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TACHYONS IN ROBERTSON–WALKER COSMOLOGY

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Superluminal signal transfer is studied in the context of a preferred cosmic frame of reference provided by the galactic background. The receding galaxies constitute a frame of absolute rest, in which the energy of tachyons (faster-than-light particles) is unambiguously defined as a positive quantity. The causality violation which arises in relativistic tachyonic theories is avoided. We define interactions of particles and tachyons in terms of elastic head-on collisions and energy-momentum conservation. To compare the theory developed with existing relativistic theories, tachyons are studied at first in a Minkowski universe, and the causality of a superluminal communication process is analyzed. Then we discuss the dynamics of tachyons in a Robertson–Walker universe with linear expansion factor and negatively curved three-space. We point out the consequences that the space expansion has on tachyons, like a finite life-time in the frame of absolute rest, and multiple images in the rest frames of moving observers.

1. Introduction

Whenever one attempts to introduce superluminal signal transfer in the context of special relativity, one is confronted with the break-down of our traditional conception of causality.[1–4] In contemporary physics it is usually taken for granted that (I) every effect has a cause, that (II) the cause precedes the effect, and that (III) the distinction of cause and effect is absolute and unambiguous.

It is very easy to see that this causality principle (I)–(III) breaks down as soon as one introduces tachyonic signal transfer in the context of special relativity.[2] Let us consider two events separated by time and space intervals $\Delta t > 0$, $\Delta x > 0$. In a co-ordinate frame (t', x') moving with relative speed $0 < u < c$, the events are separated in time by $\Delta t' = \Delta t(1 - uv/c^2)(1 - u^2/c^2)^{-1/2}$, with $v := \Delta x/\Delta t$. If we assume that these events correspond to the emission and absorption of a tachyon ($v > c$), and if we choose the relative speed u in the range $c > u > c^2/v$, then Δt and $\Delta t'$ have opposite signs. Accordingly the time order of these events is interchanged in the two frames, what is emission in the first frame is absorption in the second, and vice versa. (We call a signal emitted at space point A and absorbed at B if it moves from A to B in the proper time of the respective observer. We do not make here any specific assumptions on the energy transfer.) One of the observers will see the signal moving from A to B, and in the rest frame of the other it moves from B to A. Whatever the message is that the tachyon carries, the two observers will come to different conclusions on cause and effect, if they take for granted that the cause precedes the effect. In the context of the principle of special relativity both observations are equally real, therefore the causality principle (I)–(III) fails whenever tachyonic events are considered in special relativity.[5] The authors of Refs. 1–4 attempted to generalize this principle by dropping either the third or second condition.

In this article, we define and discuss a classical mechanics of tachyonic signals based on the traditional causality principle (I)–(III). To this end we introduce an absolute space conception. In special relativity space is the void, generated by rectangular coordinate axes. In cosmology, however, space is generated by the galactic grid, and we take this grid as the universal frame of reference.

In Sec. 2, we consider a Minkowskian universe with a static galactic distribution. We say an observer is at absolute rest if he is at rest with respect to this distribution. Observers in uniform motion will recognize their movement by seeing the galactic background passing by, Lorentz-contracted, and Doppler-shifted. Uniformly moving observers are connected to the galactic frame of absolute rest by Lorentz transformations. Uniform motion and rest are the speed of light is the same in all frames. For an observer at absolute rest the galactic grid is homogeneous, isotropic, and, in a Minkowski universe, static. To an observer in uniform motion the galactic distribution appears in motion and Lorentz contracted. Uniform motion and rest are absolute and easily distinguishable states in this context.

We define the dynamics of tachyons in the frame of absolute rest, and describe how tachyons appear to observers in moving frames. (This is done in Sec. 2 for a static, and in Sec. 7 for an expanding universe.) In Sec. 2, we discuss in detail the superluminal communication process of a galactic observer with an observer in uniform motion.

In Sec. 3, we define interactions of tachyons with subluminal classical point-particles. We assume elastic head-on collisions, which are entirely determined by energy-momentum conservation. The use of energy-momentum conservation to determine interactions is only then possible if the energy of the particles and tachyons involved is bounded from below. In the context of relativistic tachyonic theories this is very hard to achieve because Lorentz boosts may change the sign of tachyonic energy.[1,4] In this theory, the energy of tachyons is positive in the frame of absolute rest, and elastic head-on collisions are unambiguously defined by energy-momentum conservation.

In Secs. 4–7, we consider a Robertson–Walker (RW) cosmology with a linear expansion factor and negatively curved three-space geometry. This four-manifold is flat and maximally symmetric. Therefore symmetry transformations analogous to Lorentz boosts can be defined which synchronize the rest frames of geodesically moving observers with the frame of absolute rest. In Secs. 4 and 5, we give an explicit realization of these transformations, and study the world-lines of particles (observers) and tachyons. In comoving RW coordinates, all galaxies have constant space coordinates; this is just the meaning of comoving coordinates. The mutual recession of the galaxies is determined by the expansion factor, which defines the length scale on the three-space at a given instant of cosmic time. Observers at absolute rest have, by definition, constant space coordinates in the comoving galactic frame. Contrary to the static Minkowski universe, tachyons have only a finite life-time in this frame of absolute rest.

In Sec. 6, we define energy for tachyons in moving frames. As in Sec. 2, we use the galactic frame of absolute rest as reference frame, and carry over the definition of energy-momentum by means of the symmetry transformations derived in Secs. 4 and 5. In moving frames, energy is not any more positive definite; whenever it becomes negative this indicates a change in the time order of events. If an observer at absolute rest sees the tachyon moving from space point A to B, it may appear to the moving observer to head from B to A. (We say an observer is 'moving' or 'moving in the frame of absolute rest', if he is *not* comoving with the galactic background. We consider in this paper only geodesically moving observers. The moving observer can infer from the negative energy of the tachyon that the time order of these events is inverted (as compared to the cosmic time order in the frame of absolute rest) in his frame of rest.

This is also the topic of Sec. 7, namely how tachyons appear to observers in moving frames. As mentioned, tachyons have a finite life-time in the frame of absolute rest. They steadily loose energy until they finally disappear; this is an effect of the expansion of the three-space. In an appropriately chosen moving frame, this life-time may be arbitrarily large or small. In fact, there are frames in which the tachyon does not emerge at all. The most surprising effect of the expansion is, that double images of tachyons appear in the rest frames of moving observers. This happens whenever the time dependent energy of the tachyon undergoes a sign change in the moving frame. The tachyon will then emerge at two different space points at the same time!

Although this theory of superluminal motion is based on an absolute spacetime, it does not conflict with the standard theory of relativity for subluminal particles. This is further discussed in the conclusion (Sec. 8).

2. Tachyons and Causality in an Absolute Spacetime Conception

In order to compare with existing relativistic theories of superluminal motion,[1,2] and for technical simplicity, we assume in this section that spacetime is Minkowskian, and that the galactic background is static. The Euclidean three-space of this universe is generated by a more or less homogeneous and isotropic grid of galaxies, invariant in time. The following discussion can easily be carried over to the expanding Robertson–Walker universe discussed in Secs. 4–7.

The static galactic grid defines the frame of absolute rest labeled by coordinates (t, x); we consider in the following communication process only uniform motion along the x-axis.

At $(0,0) =: A$ an observer O_1 emits a tachyonic signal T_1 which moves with velocity $v_1 > 1$ $(c = 1)$. A second observer O_2 moves with velocity $0 < u < 1$, and is at time $t = 0$ at $x_1 > 0$. The tachyon T_1 hits observer O_2 at $(x_1(v_1 - u)^{-1}, x_1(1 + u(v_1 - u)^{-1})) =: B$. At this moment observer O_2 responds by emitting a tachyon T_2. (The terms 'emission' and 'absorption' refer to the initial and terminal points of the trajectory in the respective coordinate frames.)

The rest frame (t', x') of O_2 is connected with the frame of absolute rest by a Lorentz boost

$$t' = \gamma(t - ux + ux_1), \qquad x' = \gamma(x - x_1 - ut)\,, \tag{2.1}$$

$\gamma := (1 - u^2)^{-1/2}$, so that $(0, x_1)$ is mapped into $(0,\ 0)$. In the rest frame of O_2 we have for events A and B the coordinates $A' = (\gamma u x_1, -\gamma x_1)$ and $B' = (\gamma^{-1} x_1 (v_1 - u)^{-1}, 0)$, respectively.

In the frame (t', x') tachyon T_1 has the velocity

$$v_1' = (v_1 - u)(1 - uv_1)^{-1}\,. \tag{2.2}$$

For the space and time separation of events A' and B', we have

$$(\Delta x')_{A'B'} = \gamma x_1\,, \qquad (\Delta t')_{A'B'} = \gamma x_1 (v_1 - u)^{-1}(1 - uv_1)\,, \tag{2.3}$$

and $v_1' = (\Delta x'/\Delta t')_{A'B'}$.

If $1 < v_1 < u^{-1}$, then the time interval in (2.3) is positive, and so is v_1'. In the rest frame of O_2 tachyon T_1 moves from O_1 to O_2. If, however, $v_1 > u^{-1}$, then $(\Delta t')_{A'B'}$ is negative, likewise v_1'. In this case T_1 moves from O_2 toward O_1 in the rest frame of O_2. In the frame of absolute rest T_1 moves in either case from O_1 to O_2. Thus, if $v_1 > u^{-1}$, a time inversion occurs. In the frame of absolute rest T_1 moves from O_1 to O_2, whereas in the rest frame of O_2 it moves from O_2 to O_1; emission and absorption are interchanged in the two frames.

We attach to tachyons a positive mass and define in the frame of absolute rest energy and momentum as

$$E = \frac{m}{\sqrt{\varepsilon(1 - \mathbf{v}^2)}}\,, \qquad \mathbf{p} = \frac{m\mathbf{v}}{\sqrt{\varepsilon(1 - \mathbf{v}^2)}}\,, \tag{2.4}$$

with $\varepsilon = -1$ for tachyons, and $\varepsilon = 1$ for particles. Then we define energy and momentum in a uniformly moving frame (say, the rest frame (t', x') of observer O_2) by

$$E' := \gamma(E - up) = \frac{m\,\mathrm{sign}(1 - uv)}{\sqrt{\varepsilon(1 - v'^2)}}\,, \tag{2.5}$$

$$p' := \gamma(p - uE) = \frac{mv'\,\mathrm{sign}(1 - uv)}{\sqrt{\varepsilon(1 - v'^2)}}\,. \tag{2.6}$$

Here we use the frame of absolute rest as reference frame, and define so energy and momentum in all uniformly moving frames. (We formally regard $(E, \mathbf{p})$ defined in

(2.4) as a four-vector, and apply the relevant Lorentz transformation. In (2.5) and (2.6), we consider only one space dimension for simplicity, and v' is related to v via the addition law (2.2) for velocities.) The energy in the frame of absolute rest is always positive, but E' need not be.

Remark: For particles, Eqs. (2.4) are an invariant definition of energy and momentum, valid in all uniformly moving frames in Minkowski space. However, this is not true for tachyons. We may define a Lagrangian for freely moving particles/tachyons as

$$L = -m\sqrt{-\varepsilon\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}\,, \tag{2.7}$$

with $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$, and $x^\mu = (t, \mathbf{x})$. Then we choose a parametrization $x^\mu(s)$ so that $\dot{t}^2 - \dot{\mathbf{x}}^2 = \varepsilon$ and $\dot{t} > 0$. Finally we define $p^\mu = (E, \mathbf{p}) = m\dot{x}^\mu$, which coincides with (2.4). If $\varepsilon = 1$, then proper orthochronous Lorentz transformations do not change the sign of E, but they may if $\varepsilon = -1$. This is evident from (2.5), since we always have $\mathrm{sign}(1-uv) = 1$ for particles, but not necessarily for tachyons. Therefore a frame of reference is needed to unambiguously define the sign of tachyonic energy in every frame. In cosmology this frame is provided by the galactic background.

Let us come back to the discussion of Eq. (2.3). If $1 < v_1 < u^{-1}$, the energy of tachyon T_1 is positive in the rest frame of O_2 (and, by definition, in the rest frame of O_1). Both observers see T_1 moving from O_1 to O_2. Observer O_1 looses energy by emitting T_1, and O_2 gains energy by absorbing the tachyon. In both frames the loss of energy by O_1 happens prior to the gain of energy by O_2.

If $v_1 > u^{-1}$, the energy of T_1 is negative in the rest frame of O_2. Also in this case O_1 looses and O_2 gains energy in both frames. In the rest frame of O_2, however, the gain of energy by O_2 happens prior to the loss of energy by O_1, since T_1 has negative energy and moves from O_2 to O_1. This time inversion is a consequence of the observer's movement in the frame of absolute rest. However, O_2 can infer from the galactic background passing by, or from the negative energy of T_1 that the cosmic time order is distorted in his rest frame. [Negative energy always indicates a change of the time order, cf. (2.3) and (2.5).] He will conclude that in the frame of absolute rest the loss of energy by O_1 (cause) happens prior to the gain of energy by O_2 (effect). Thus both O_1 and O_2 come to the same conclusions in accordance with the causality principle, cf. Sec. 1.

Next we study the second part of the communication process, namely the response of O_2, who emits a tachyon T_2 as soon as he absorbed T_1 at B'. In the frame of absolute rest (t, x) the trajectory of T_2 is

$$x_T(t) = x_1(1 + u(v_1 - u)^{-1}) + v_2(t - x_1(v_1 - u)^{-1})\,. \tag{2.8}$$

v_2 is the velocity of T_2 in the frame of absolute rest; it may also be subluminal in the following discussion. In the rest frame (t', x') of observer O_2 the trajectory of T_2 reads as

$$x'_T(t') = (v_2 - u)(1 - v_2 u)^{-1}(t' - \gamma^{-1}x_1(v_1 - u)^{-1})\,, \tag{2.9}$$

and the trajectory of observer O_1 reads

$$x'_O(t') = -ut' - \gamma^{-1}x_1\,. \tag{2.10}$$

Observer O_1 and tachyon T_2 collide in the rest frame of O_2 at

$$\begin{aligned} t'_{\mathrm{coll}} &= \gamma^{-1}x_1(v_1-u)^{-1}(1-\gamma^2 v_1 v_2^{-1}(1-v_2u))\,,\\ x'_{\mathrm{coll}} &= -\gamma x_1 v_1 v_2^{-1}(v_2-u)(v_1-u)^{-1}\,. \end{aligned} \tag{2.11}$$

The collision happens after the emission in this frame if

$$t'_{\mathrm{coll}} > \gamma^{-1}x_1(v_1-u)^{-1} =: t'_{\mathrm{em}}\,. \tag{2.12}$$

t'_{em} is the time at which T_2 is emitted in the rest frame of O_2. Condition (2.12) is only satisfied if $v_2 < 0$ or $v_2 > u^{-1}$.

In the frame of absolute rest the collision of T_2 with O_1 takes place at

$$t_{\mathrm{coll}} = x_1(v_1-u)^{-1}(1-\gamma^2 v_1 v_2^{-1})\,, \qquad x_{\mathrm{coll}} = 0\,. \tag{2.13}$$

The collision happens after the emission,

$$t_{\mathrm{coll}} > x_1(v_1-u)^{-1} =: t_{\mathrm{em}}\,, \tag{2.14}$$

only if $v_2 < 0$. Then v'_2 is in the range $[-u^{-1}, -u]$, according to (2.2).

If $v_2 < 0$, tachyon T_2 collides with O_1 at t_{coll} ($> t_{\mathrm{em}}$). In the rest frame of O_2 it collides with O_1 at t'_{coll} ($> t'_{\mathrm{em}}$). If $0 < v_2 < u^{-1}$, T_2 does not collide with O_1 in either frame.

If $v_2 > u^{-1}$, the energy of T_2 is negative in the rest frame of O_2, which indicates time inversion (compare the discussion of T_1 with $v_1 > u^{-1}$). In the frame of absolute rest we have $t_{\mathrm{coll}} < t_{\mathrm{em}}$, which means that T_2 is actually emitted by O_1 at t_{coll} and absorbed by O_2 at t_{em}. In the rest frame of O_2 the tachyon moves with negative energy from O_2 to O_1 in the time interval $[t'_{\mathrm{em}}, t'_{\mathrm{coll}}]$, $t'_{\mathrm{coll}} > t'_{\mathrm{em}}$. Observer O_2 concludes from the negative energy of the tachyon that the cosmic time order is inverted in his frame of rest. He cannot emit a tachyon of negative energy.

Remarks:

(1) Emission and absorption are defined in a frame dependent way without reference to energy transfer. We say that in a given rest frame, a tachyon is emitted at space point A and absorbed at B, if A is the initial and B the terminal point of its trajectory in this frame. The trajectory is of course parametrized by the time of the respective rest frame. Emission and absorption have nevertheless an absolute meaning. A signal is emitted at A and absorbed at B if this is the case in the frame of absolute rest, where the trajectory is parametrized by cosmic time.

(2) If a tachyon is emitted by a moving observer, this emission must also appear as such in his rest frame, otherwise the tachyon would emerge in his rest frame before he creates it, and his action would be predetermined. Accordingly, a moving observer can only emit tachyons of positive energy in his rest frame. This 'observer' may be for example, a decaying classical particle. In the next section we study elastic head-on collisions of particles and tachyons. The decay of a classical particle into a particle and a tachyon is a special case of that, we just have to put the mass of the incoming tachyon zero. If, however, a tachyon is absorbed by a moving observer, this absorption may well appear as emission in his rest frame; after all, the emission he observed is not caused by him.

Let us summarize this communication process. In the frame of absolute rest a tachyon T_1 is emitted by O_1 at $t = 0$. If its velocity v_1 is in the range $1 < v_1 < u^{-1}$, then its energy is positive in the rest frame of O_2, and both observers see T_1 moving from O_1 to O_2. If $v_1 > u^{-1}$, then the energy of T_1 is negative in the rest frame of O_2, so that emission and absorption are interchanged compared to the frame of absolute rest. Also in this case energy is transferred from O_1 to O_2 in both frames, but observer O_2 sees this process time inverted, namely his gain of energy prior to the loss of energy by O_1. That his observation is time inverted he can infer from the negative energy of the tachyon and from his own motion in the frame of absolute rest. Thus both observers come to the same conclusion, the gain of energy by O_2 is caused by the energy loss of O_1, and the cause precedes the effect.

Upon receipt of the signal, O_2 emits a tachyon T_2 with $v_2 < 0$. (v_2 is the velocity of T_2 in the frame of absolute rest; if v_2 is in the range $0 < v_2 < u^{-1}$, the tachyon cannot reach O_1, and if $v_2 > u^{-1}$, it cannot be emitted by O_2.) The energy of T_2 is positive in the rest frame of O_2, and therefore both observers see the tachyon moving from O_2 to O_1. Tachyon T_2 can reach O_1 only after the emission of T_1, thus no predetermination can arise. The whole process complies with the causality principle (I)–(III) formulated at the beginning of Sec. 1.

3. Tachyon-Particle Collisions in the Universal Cosmic Frame

We study tachyon-particle interactions, elastic head-on collisions, which are entirely determined by energy-momentum conservation. So we do not have to assume any specific interaction mechanism. There is an important difference to elastic particle–particle collisions. As we have seen in Sec. 2, the sign of tachyonic energy is not preserved under Lorentz boosts. Tachyonic energy, as an interaction parameter, must be bounded from below, otherwise one could extract an infinite amount of energy from a tachyon. Energy is positive definite only in the frame of absolute rest. In this frame we will determine now, by energy–momentum conservation, interaction processes of tachyons with particles.

$E_{p,t}, m_{p,t}$ denote energy and mass (always positive) of ingoing particle and tachyon. p and t are particle and tachyon momenta; $\tilde{E}_{p,t}, \tilde{p}$, and $\tilde{t}$ are the respective quantities for the outgoing particle and tachyon.

The interaction is then based on the conservation laws

$$\tilde{E}_t + \tilde{E}_p = E_t + E_p\,, \qquad \tilde{p} + \tilde{t} = p + t\,. \tag{3.1}$$

From (2.4), we have

$$-E_p^2 + p^2 = -m_p^2 = -\tilde{E}_p^2 + \tilde{p}^2\,, \quad -E_t^2 + t^2 = m_t^2 = -\tilde{E}_t^2 + \tilde{t}^2\,. \tag{3.2}$$

Apart from the trivial solution $(E_p, p, E_t, t) = (\tilde{E}_p, \tilde{p}, \tilde{E}_t, \tilde{t})$ we have as the only solution of Eqs. (3.1) and (3.2)

$$\tilde{E}_p = E_p - (p+t)\lambda\,, \qquad \tilde{p} = p - (E_t + E_p)\lambda\,, \tag{3.3}$$

$$\tilde{E}_t = E_t + (p+t)\lambda\,, \qquad \tilde{t} = t + (E_t + E_p)\lambda\,, \tag{3.4}$$

with $\lambda := 2(tE_p - pE_t)[-(E_p + E_t)^2 + (p+t)^2]^{-1}$. Moreover, $\lambda(E_p, p, E_t, t) = -\lambda(\tilde{E}_p, \tilde{p}, \tilde{E}_t, \tilde{t})$.

Using (3.2), we express p^2, t^2 in (3.3) and (3.4) by $E_{p,t}, m_{p,t}$, and obtain so for the outgoing energies and momenta

$$\tilde{E}_p = \frac{-E_p(m_t^2 + m_p^2) + 2E_t(tp - m_p^2) - 2E_t^2 E_p}{m_t^2 - m_p^2 + 2tp - 2E_t E_p}\,, \tag{3.5}$$

$$\tilde{p} = \frac{p(m_t^2 - m_p^2 + 2E_t^2) - 2t(m_p^2 + E_p E_t)}{m_t^2 - m_p^2 + 2tp - 2E_t E_p}\,, \tag{3.6}$$

$$\tilde{E}_t = \frac{E_t(m_t^2 + m_p^2) + 2E_p(tp + m_t^2) - 2E_p^2 E_t}{m_t^2 - m_p^2 + 2tp - 2E_t E_p}\,, \tag{3.7}$$

$$\tilde{t} = \frac{t(m_t^2 - m_p^2 + 2E_p^2) + 2p(m_t^2 - E_p E_t)}{m_t^2 - m_p^2 + 2tp - 2E_t E_p}\,, \tag{3.8}$$

with $p = \operatorname{sign}(p)\sqrt{E_p^2 - m_p^2}$ and $t = \operatorname{sign}(t)\sqrt{E_t^2 + m_t^2}$. If $\operatorname{sign}(pt) = 1$, the momenta evidently have the same orientation.

In the frame of absolute rest, energy and momentum can only be transferred if the energy of the outgoing particle is larger than its rest mass, and the outgoing tachyon energy is positive, cf. (2.4),

$$\tilde{E}_p \geq m_p\,, \quad \tilde{E}_t \geq 0\,. \tag{3.9}$$

Therefore, even if particle and tachyon are geometrically colliding, we must assume the trivial solution of Eqs. (3.1) and (3.2), i.e., no energy-momentum transfer if conditions (3.9) are not satisfied. In the following we will determine conditions for the ingoing particle and tachyon energies and momenta, so that solutions (3.5) and (3.7) satisfy (3.9); only then interaction is possible. This is in strong contrast to elastic particle-particle collisions, where the positivity of the outgoing energies is already ensured by the positivity of the incoming energies.

At first we study the limit behavior of solutions (3.5) and (3.7) for $E_t \to 0$ (i.e., tachyonic speed to infinity). We obtain asymptotically

$$\begin{aligned} \tilde{E}_p &\sim \frac{-E_p(m_t^2 + m_p^2)}{m_t^2 - m_p^2 - 2m_t\sqrt{E_p^2 - m_p^2}}\,, \\ \tilde{E}_t &\sim \frac{2E_p(\mathrm{sign}(pt)m_t\sqrt{E_p^2 - m_p^2} + m_t^2)}{m_t^2 - m_p^2 + 2m_t\,\mathrm{sign}(pt)\sqrt{E_p^2 - m_p^2}}\,. \end{aligned} \tag{3.10}$$

Conditions (3.9) are satisfied in this limit if and only if $\mathrm{sign}(pt) = -1$ and $|p| \geq m_t$. A short inspection of (3.5) and (3.7) tells us, that under these conditions interaction is possible in the whole range $0 \leq E_t < \infty$, since (3.9) then always holds. If a tachyon with velocity $|\mathbf{v}_t| \to \infty$ collides head-on with a particle whose momentum supersedes the tachyonic mass, then energy is transferred from the particle to the tachyon, and both particle and tachyon slow down.

Remarks: (1) Eqs. (3.10) indicate that a particle can emit a tachyon of a rest mass lower than the particle momentum. (2) If $p = -t$, the frame of absolute rest also happens to be the center-of-mass system, and interaction is always possible. (3) Since the positivity of tachyonic energy is not preserved under Lorentz boosts, it is not advisable to study these interaction processes in the rest frame of the particle.

Next we consider the limit $E_t \to \infty$, namely collisions of a slow tachyon ($|\mathbf{v}_t| \to 1$) with a particle. We have in this limit

$$\tilde{E}_p \sim E_t\,, \qquad \tilde{E}_t \sim \frac{1}{2}\frac{m_t^2 + m_p^2 + 2\,\mathrm{sign}(pt)E_p\sqrt{E_p^2 - m_p^2} - 2E_p^2}{\mathrm{sign}(pt)\sqrt{E_p^2 - m_p^2} - E_p}\,. \tag{3.11}$$

We assume here of course that $\tilde{E}_p$ is larger than the rest mass m_p. $\tilde{E}_t$ is positive if and only if $\mathrm{sign}(pt) = -1$ and $|p| \geq \frac{1}{2}m_t^{-1}(m_t^2 - m_p^2)$, or $\mathrm{sign}(pt) = 1$ and $|p| \leq \frac{1}{2}m_t^{-1}(m_p^2 - m_t^2)$. In this limit, $E_t \to \infty$, energy is transferred to the particle, and both particle and tachyon accelerate. A particle at rest can interact with the tachyon if $m_p > m_t$, provided E_t is sufficiently large, cf. (3.13).

For $E_p \to \infty$, we have the asymptotic limits

$$\tilde{E}_p \sim \frac{-(m_t^2 + m_p^2) + 2E_t|t|\mathrm{sign}(pt) - 2E_t^2}{2|t|\mathrm{sign}(pt) - 2E_t}\,, \qquad \tilde{E}_t \sim |p|\,. \tag{3.12}$$

Interaction is possible if and only if $\mathrm{sign}(pt) = -1$. Energy is transferred from the particle to the tachyon if the energy of the incoming tachyon is small.

Finally, for $E_p \to m_p$, Eqs. (3.5) and (3.7) have the limits

$$\begin{aligned} \tilde{E}_p &\sim \frac{-m_p(m_t^2 + m_p^2) + 2E_t m_p + 2E_t^2)}{m_t^2 - m_p^2 - 2E_t m_p}\,, \\ \tilde{E}_t &\sim \frac{E_t(m_t^2 - m_p^2) + 2m_p m_t^2}{m_t^2 - m_p^2 - 2E_t m_p}\,. \end{aligned} \tag{3.13}$$

Interaction is possible if and only if $m_p > m_t$, and $E_t \geq 2m_p m_t^2(m_p^2 - m_t^2)^{-1}$. Tachyonic energy is transferred to the particle; if E_t is large, this happens to a substantial degree.

Remarks:

(1) In the rest frame of a moving observer tachyonic energy need not be positive, as pointed out in Sec. 2. However, even in a moving frame it is not possible to extract an arbitrarily large amount of energy from a tachyon (by means of some, not necessarily elastic, particle-tachyon interaction). In fact, if in the rest frame of a moving observer the energy of the tachyon is negative and arbitrarily large, then its velocity in this frame would be arbitrarily close to the speed of light. The same then holds true for its velocity in the frame of absolute rest, cf. (2.2). This means that the energy (2.5) cannot be negative, contrary to our assumption. This reasoning can easily be made quantitative; there is a lower bound on the energy of the tachyon in every frame.

(2) We have formulated here the collision laws in the static Minkowski universe introduced in Sec. 2. All considerations carry over to the Robertson–Walker geometry discussed in the following sections. We then consider collisions along the t-semiaxis of the hyperbolic half-space H^3, cf. Sec. 4 for definitions. The only change we have to make in the formulas of this section is to rescale the squared momenta, $p^2 \to p^2 a^2(\tau) t^{-2}$, and the same for tachyon momenta. This is because $\mathbf{p}^2 = \gamma_{ij} p^i p^j = \gamma_{33} p^2$ along the t-axis, with γ_{ij} the metric on the spacelike slices. The same procedure has to be performed with the velocity squares $\mathbf{v}_{p,t}^2$ in formulas (2.4).

4. Symmetry Transformations in an Expanding, Flat Universe

In the following sections, we will study tachyons in a Robertson–Walker (RW) cosmology with negatively curved, open three-space, and linear expansion factor. We will derive a well defined energy for tachyons, based on symmetry transformations which synchronize the coordinate frames of geodesically moving observers with the frame of absolute rest. In this section, we give an explicit representation of these symmetries of the RW line element.

The cosmology we consider is based on the line element

$$ds^2 = -c^2 d\tau^2 + a^2(\tau) d\sigma^2 \,, \qquad d\sigma^2 := R^2 t^{-2}(|dz|^2 + dt^2) \,, \tag{4.1}$$

with expansion factor $a(\tau) := cR^{-1}\tau$, $0 < \tau < \infty$. $d\sigma^2$ is the line element of the Poincaré half-space[6] H^3 [rectangular coordinates (z, t), $z := x + iy$, $t > 0$, and curvature $-1/R^2$]. In this paper we put $R = c = 1$.

This four-manifold is flat, and can isometrically be mapped onto the interior of the forward light cone. The Minkowski line element

$$d\tilde{s}^2 = -d\lambda^2 + |d\omega|^2 + d\rho^2 \,, \tag{4.2}$$

$\omega := x_1 + ix_2$, is transformed by

$$\lambda = \frac{1}{2}\tau t^{-1}(|z|^2 + t^2 + 1)\,, \qquad \omega = \tau t^{-1} z\,, \qquad \rho = \frac{1}{2}\tau t^{-1}(|z|^2 + t^2 - 1)\,, \quad (4.3)$$

into ds^2. We immediately see that

$$\rho - \lambda = -\tau t^{-1} < 0\,, \qquad |\omega|^2 + \rho^2 - \lambda^2 = -\tau^2 < 0\,, \qquad \lambda > 0\,. \quad (4.4)$$

For the geometric meaning of this transformation and the way to construct it, see Ref. 7. Inversely, we have

$$\tau = \sqrt{\lambda^2 - |\omega|^2 - \rho^2}\,, \qquad z = \frac{\omega}{\lambda - \rho}\,, \qquad t = \frac{1}{\lambda - \rho}\sqrt{\lambda^2 - |\omega|^2 - \rho^2}\,, \quad (4.5)$$

with λ, ω, ρ ranging in the forward light cone defined in (4.4).

By the diffeomorphism (4.3), the Poincaré group is also the invariance group of the line element (4.1), its action in the RW geometry is given by

$$(\tau, z, t) \to (\lambda, \omega, \rho) \to \Lambda(\lambda, \omega, \rho) = (\lambda', \omega', \rho') \to (\tau', z', t')\,. \quad (4.6)$$

Λ denotes here the standard representation of the Poincaré group in Minkowski space.

We consider at first the proper orthochronous Lorentz group $SO^+(3,1)$, which leaves the forward light cone invariant. Its action in the RW geometry, obtained from (4.6), is[8]

$$\tau' = \tau\,, \qquad (z', t') = \frac{1}{|cz + d|^2 + |c|^2 t^2}((az + b)(\overline{cz + d}) + a\bar{c}t, t)\,, \quad (4.7)$$

with $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ a complex matrix of determinant one. Note that (4.7) is invariant under a sign change of the coefficients a, b, c, d, and that $SO^+(3,1)$ is isomorphic to $SL(2, C)/\{\pm \mathrm{id}\}$. The derivation of (4.7) from the sequence (4.6) just amounts to an explicit realization of this well known isomorphism.

The Lorentz boosts, which mix space and time in Minkowski space, do not affect cosmic time at all; $SO^+(3,1)$ is the invariance group of the spacelike sections. The spacetime mixing symmetry transformations of the line element (4.1) are generated by spacetime translations in Minkowski space. On that we will have a closer look now.

We consider the shift

$$\lambda' = \lambda - h\,, \qquad \omega' = \omega\,, \qquad \rho' = \rho - b\,, \quad (4.8)$$

with real numbers b, h. We suppress here a shift of ω, for it can be generated by combining the transformations (4.8) with $SO^+(3,1)$-transformations.

The action of the shift (4.8) in the RW geometry is easily calculated, cf. (4.3) and (4.5),

$$\tau' = t^{-1/2}\sqrt{[\tau + t(b-h)][\tau t - (b+h)] + \tau|z|^2(b-h)}\,, \tag{4.9}$$

$$z' = \frac{\tau z}{\tau + t(b-h)}\,, \tag{4.10}$$

$$t' = \frac{t^{1/2}}{\tau + t(b-h)}\sqrt{[\tau + t(b-h)][\tau t - (b+h)] + \tau|z|^2(b-h)}\,. \tag{4.11}$$

As a consequence of (4.9)–(4.11), we have the identities

$$\begin{aligned} &\tau + t(b-h) = \frac{t\tau'}{t'}\,, && \tau t - (b+h) = \tau' t' - (b-h)|z|^2\frac{\tau t'}{\tau' t}\,,\\ &\frac{\tau}{t}\left(\frac{d\tau}{\tau} - \frac{dt}{t}\right) = \frac{\tau'}{t'}\left(\frac{d\tau'}{\tau'} - \frac{dt'}{t'}\right), && \frac{z'\tau'}{t'} = \frac{z\tau}{t}\,. \end{aligned} \tag{4.12}$$

In Sec. 6, we will need the differential version of the transformation (4.9)–(4.11),

$$\begin{aligned} \frac{d\tau'}{\tau'} &= \frac{\tau}{2tt'\tau'}\left\{\frac{2t'}{\tau'}(b-h)(xdx + ydy) + \frac{d\tau}{\tau}\left[t^2 + t'^2 + (b-h)^2\frac{t'^2}{\tau'^2}|z|^2\right]\right.\\ &\quad \left. + \frac{dt}{t}\left[t^2 - t'^2 - (b-h)^2\frac{t'^2}{\tau'^2}|z|^2\right]\right\},\\ \frac{dt'}{t'} &= \frac{\tau}{2tt'\tau'}\left\{\frac{2t'}{\tau'}(b-h)(xdx + ydy) + \frac{d\tau}{\tau}\left[t^2 - t'^2 + (b-h)^2\frac{t'^2}{\tau'^2}|z|^2\right]\right.\\ &\quad \left. + \frac{dt}{t}\left[t^2 + t'^2 - (b-h)^2\frac{t'^2}{\tau'^2}|z|^2\right]\right\},\\ dz' &= \frac{t'}{\tau'}\left\{\frac{\tau}{t}dz + z'(b-h)\left[\frac{d\tau}{\tau} - \frac{dt}{t}\right]\right\}, \end{aligned} \tag{4.13}$$

which can easily be calculated using Eqs. (4.12).

Remarks:

(1) Unlike $SO^+(3,1)$-transformations, the shifts (4.8) do not leave the forward light cone invariant. The physical consequences of that we will discuss in the next sections. The domain of definition of a shift depends on the values of the shift parameters b and h. The domain

$$\tau > 0, t > 0, \tau + t(b-h) > 0, [\tau + t(b-h)][\tau t - (b+h)] + \tau|z|^2(b-h) > 0\,, \tag{4.14}$$

is bijectively mapped by the transformation (4.9)–(4.11) onto the domain

$$\tau' > 0, t' > 0, \tau' - t'(b-h) > 0, [\tau' - t'(b-h)][\tau' t' + (b+h)] - \tau'|z'|^2(b-h) > 0\,. \tag{4.15}$$

(2) The inverse of the map (4.9)–(4.11) is obtained by replacing (b, h) by $(-b, -h)$. Likewise, Eqs. (4.12) remain true if we interchange (τ, z, t) and (τ', z', t'), and replace (b, h) by $(-b, -h)$.

(3) The map (4.9)–(4.11) is by construction a symmetry of the line element (4.1), and the analogue to a Lorentz boost in Minkowski space. We will show in Sec. 5 that a particle (observer) geodesically moving along the t-axis can be mapped via (4.9)–(4.11) into its rest frame. The t-axis is in no way distinguished apart from technical simplicity. Since H^3 is a homogeneous space, the trajectory of a geodesically moving particle/ray/tachyon can always be mapped via a suitable transformation (4.7) into the t-axis. As pointed out in Remark (1), the transformations (4.9)–(4.11) are not global isometries of the RW geometry, contrary to Lorentz boosts in Minkowski space. Their domains of definition are given in (4.14). In the next section we will obtain by means of them rest frames for geodesically moving observers.

5. World-Lines and Rest Frames

We define at first sub- and super-luminal geodesic motion in the RW cosmology introduced in Sec. 4. The action reads

$$S = \int L ds\,, \qquad L = -m\sqrt{-\varepsilon g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}\,, \tag{5.1}$$

with $\varepsilon = 1$ for particles, and $\varepsilon = -1$ for tachyons. $x^\mu = (\tau, z, t)$. The metric $g_{\mu\nu}$ is defined in (4.1) [$g_{00} = -1$, $g_{ij} = a^2(\tau)t^{-2}\delta_{ij}$, $g_{0i} = 0$]. We assume, without loss of generality [cf. Remark (3) after Eq. (4.13)], that the particle/tachyon moves along the t-axis, $z = 0$, of H^3.

We obtain as a first integral of motion

$$\dot{\tau}^2(s) - a^2(\tau)t^{-2}\dot{t}^2(s) = \varepsilon\,. \tag{5.2}$$

Since $\ln t$ is a cyclic variable, there is a second integral,

$$a^2(\tau)t^{-1}\dot{t} = \nu_{p,t}\,, \tag{5.3}$$

with a real integration constant $\nu_{p,t}$. Combining (5.2) and (5.3), we obtain

$$\dot{\tau}^2 = \varepsilon + \nu_{p,t}^2 a^{-2}(\tau)\,, \qquad \frac{1}{t}\frac{dt}{d\tau} = \pm\frac{\nu_{p,t}}{a(\tau)\sqrt{\varepsilon a^2(\tau) + \nu_{p,t}^2}}\,. \tag{5.4}$$

If we put $\varepsilon = 0$ in (5.2) and (5.4), we obtain the equations for light rays. From (5.4) we have for the absolute value of the velocity

$$|\mathbf{v}| = \left|\frac{dt}{d\tau}\right|\frac{a(\tau)}{t} = \frac{1}{\sqrt{1 + \varepsilon\nu_{p,t}^{-2}a^2(\tau)}}\,. \tag{5.5}$$

From now on we specify $a(\tau) = \tau$. We discuss the case $\varepsilon = 1$ (particles, observers) first. Integrating Eqs. (5.2) and (5.3), we obtain

$$\tau(s) = \sqrt{s^2 - \nu_p^2}\,, \qquad t(s) = \kappa_p \left(\frac{s - \nu_p}{s + \nu_p} \right)^{1/2}, \tag{5.6}$$

$$t(\tau) = \kappa_p \left[\frac{-\nu_p + \sqrt{\tau^2 + \nu_p^2}}{\nu_p + \sqrt{\tau^2 + \nu_p^2}} \right]^{1/2} ; \tag{5.7}$$

κ_p is a positive integration constant. The curve parameter ranges in $\infty > s > |\nu_p|$. All other geodesics (of particles) can be obtained by applying the symmetry group (4.7) of the spacelike slices to the trajectories (5.6). For light rays we have instead of (5.7) $t(\tau) = \kappa_l \tau^{\pm 1}$, κ_l a positive constant.

For tachyons, $\varepsilon = -1$, we obtain from (5.2)–(5.4),

$$\tau(s) = \sqrt{\nu_t^2 - s^2}\,, \qquad t(s) = \kappa_t \left(\frac{\nu_t + s}{\nu_t - s} \right)^{1/2}, \tag{5.8}$$

$$t(\tau) = \kappa_t \left[\frac{\nu_t - \sqrt{\nu_t^2 - \tau^2}}{\nu_t + \sqrt{\nu_t^2 - \tau^2}} \right]^{1/2} . \tag{5.9}$$

The parameter range is now $-|\nu_t| < s < 0$. κ_t is a positive integration constant. Evidently the tachyon can only exist within the period $\tau < |\nu_t|$.

By means of the transformation (4.9)–(4.11) we can easily transform a particle (observer) to rest. The observer moves along the t-axis. Therefore, to determine the parameters b, h in the map (4.9)–(4.11), we may put $z = z' = 0$. If (τ', t') denote the coordinates of the rest frame, we must have there $\tau' = s$ and $t' = \text{const}$, cf. (5.2). We insert $\tau' = s$ and the particle trajectory (5.6) into Eq. (4.9). [In (5.6), we rename the integration constants, $(\kappa_p, \nu_p) \to (\kappa_0, \nu_0)$, to distinguish between particles and observers in the next sections.] Then we determine b and h so that Eq. (4.9) is identically satisfied for all s. There is a unique solution,

$$b + h = -\nu_0 \kappa_0\,, \qquad b - h = -\nu_0 \kappa_0^{-1}\,. \tag{5.10}$$

From (4.11), we finally obtain $t' = \kappa_0$. The transformation (4.9)–(4.11) together with the coefficients (5.10) defines the rest frame (τ', t') of the observer (ν_0, κ_0) moving along the t-axis according to Eq. (5.7) [with (κ_p, ν_p) replaced by (κ_0, ν_0)].

In the following sections it will be sufficient to consider the (τ', t')-plane, and to put $z = z' = 0$, $dz = dz' = 0$ in formulas (4.9)–(4.15). Then the map (4.9)–(4.11) with b and h as in (5.10) reduces to

$$\tau' = t^{-1/2} (\tau - t \nu_0 \kappa_0^{-1})^{1/2} (\tau t + \nu_0 \kappa_0)^{1/2}\,, \tag{5.11}$$

$$t' = t^{1/2} (\tau - t \nu_0 \kappa_0^{-1})^{-1/2} (\tau t + \nu_0 \kappa_0)^{1/2}\,, \tag{5.12}$$

and the differentials (4.13) read as

$$d\tau' = \tau t^{-1/2}(\tau - t\nu_0\kappa_0^{-1})^{-1/2}(\tau t + \nu_0\kappa_0)^{-1/2}\left(\frac{d\tau}{\tau}A_+ + \frac{dt}{t}A_-\right), \quad (5.13)$$

$$dt' = \tau t^{1/2}(\tau - t\nu_0\kappa_0^{-1})^{-3/2}(\tau t + \nu_0\kappa_0)^{-1/2}\left(\frac{d\tau}{\tau}A_- + \frac{dt}{t}A_+\right),$$

$$A_+ := \tau t - \frac{1}{2}\nu_0(t^2\kappa_0^{-1} - \kappa_0), \qquad A_- := -\frac{1}{2}\nu_0(t^2\kappa_0^{-1} + \kappa_0). \quad (5.14)$$

By means of these differentials we can define energy and momentum in moving frames.

6. Tachyon Energy in an Expanding Spacetime

We consider particles and tachyons with world-lines as defined in Eqs. (5.6)–(5.9). In the frame of absolute rest we define their energy and momentum as

$$E = m\dot{\tau}, \qquad p = m\dot{t}, \quad (6.1)$$

according to the relativistic prescription $p^\mu = m\dot{x}^\mu$, cf. the Remark after (2.6). It is understood that $\dot{\tau}$ is positive, so we choose the positive root in (5.4). Using Eqs. (5.3) and (5.4), we may write

$$E = m\sqrt{\varepsilon + \nu_{p,t}^2 a^{-2}(\tau)}, \qquad p = m\nu_{p,t} t a^{-2}(\tau). \quad (6.2)$$

We then have $-E^2 + |p|^2 = -m^2\varepsilon$, with $|p|^2 := a^2(\tau)t^{-2}p^2$. From now on we specify $a(\tau) = \tau$, so that the energy in the frame of absolute rest is $E = m\tau^{-1}\sqrt{\varepsilon\tau^2 + \nu_{p,t}^2}$.

Let us consider an observer moving according to (5.6) along the t-axis. His integration parameters are (κ_0, ν_0) as in Sec. 5. His rest frame (primed coordinates) is related to the frame of absolute rest via (5.11) and (5.12). We now define the energy of particles and tachyons in the rest frame of this observer.

For particles this is straightforward to do, because (E, p) as defined in (6.1) and (6.2) transforms as a contravariant two-vector under the transformation (5.11) and (5.12). In the frame of absolute rest the particle (κ_p, ν_p) has the trajectory (5.6). In the rest frame of the moving observer this trajectory reads as

$$\tau'(s) = [s - (\nu_p - \nu_0\kappa_0\kappa_p^{-1})]^{1/2}[s + (\nu_p - \nu_0\kappa_p\kappa_0^{-1})]^{1/2}, \quad (6.3)$$

$$t'(s) = \kappa_p[s - (\nu_p - \nu_0\kappa_0\kappa_p^{-1})]^{1/2}[s + (\nu_p - \nu_0\kappa_p\kappa_0^{-1})]^{-1/2}. \quad (6.4)$$

Dividing Eqs. (5.13) and (5.14) by ds, and inserting (E, p) as defined in (6.1) and (5.6), we obtain energy and momentum of the particle in the rest frame of the observer (κ_0, ν_0) as

$$E' = m\tau'^{-1}\left[s + \frac{1}{2}\nu_0(\kappa_0\kappa_p^{-1} - \kappa_p\kappa_0^{-1})\right] = m\dot{\tau}', \quad (6.5)$$

$$p' = \frac{\nu_0 m\kappa_p}{2\tau'^2}\frac{[s - (\nu_p - \nu_0\kappa_0\kappa_p^{-1})]^{1/2}}{[s + (\nu_p - \nu_0\kappa_p\kappa_0^{-1})]^{1/2}}(2\nu_p\nu_0^{-1} - \kappa_0\kappa_p^{-1} - \kappa_p\kappa_0^{-1}) = m\dot{t}'. \quad (6.6)$$

Defining

$$\nu_p' := \nu_p - \frac{1}{2}\nu_0(\kappa_0\kappa_p^{-1} + \kappa_p\kappa_0^{-1})\,, \tag{6.7}$$

we obtain from (6.3)

$$\sqrt{\tau'^2 + \nu_p'^2} = s + \frac{1}{2}\nu_0(\kappa_0\kappa_p^{-1} - \kappa_p\kappa_0^{-1})\,. \tag{6.8}$$

Thus we have

$$E' = m\tau'^{-1}\sqrt{\tau'^2 + \nu_p'^2}\,, \tag{6.9}$$

$$p' = m\nu_p'\tau'^{-2}t'(\nu_p', \tau')\,, \tag{6.10}$$

with $t'(\nu_p', \tau')$ as in (5.7) ($(\tau, t, \nu_p) \to (\tau', t', \nu_p')$, and $\kappa_p' = \kappa_p$).

In the case of tachyons we have to put $\varepsilon = -1$ in (6.2), and then (E, p) does not any more transform as a vector, cf. Sec. 2. [The coordinate change (5.11) and (5.12) is orthochronous only for timelike vectors; if $\varepsilon = -1$, it may change the sign of the energy.] However, we use the frame of absolute rest as reference frame, and define energy and momentum in moving frames by formally regarding (E, p) as a contravariant vector. Only by means of a reference frame one can obtain a well defined sign of tachyonic energy in moving frames.

In the (τ', t')-frame, the tachyon trajectory (5.8) reads, cf. (5.11) and (5.12),

$$\tau'(s) = [\mathrm{sign}(\nu_t)s + (|\nu_t| + \nu_0\kappa_0\kappa_t^{-1})]^{1/2}[-\mathrm{sign}(\nu_t)s + (|\nu_t| - \nu_0\kappa_t\kappa_0^{-1})]^{1/2}\,, \tag{6.11}$$

$$t'(s) = \kappa_t[\mathrm{sign}(\nu_t)s + (|\nu_t| + \nu_0\kappa_0\kappa_t^{-1})]^{1/2}[-\mathrm{sign}(\nu_t)s + (|\nu_t| - \nu_0\kappa_t\kappa_0^{-1})]^{-1/2}\,. \tag{6.12}$$

We define energy and momentum along this trajectory by transforming (E, p) [given by (6.1) and (5.8) in the frame of absolute rest] via (5.13) and (5.14):

$$E' := -m\tau'^{-1}\left[s + \frac{1}{2}\mathrm{sign}(\nu_t)\nu_0(\kappa_t\kappa_0^{-1} + \kappa_0\kappa_t^{-1})\right], \tag{6.13}$$

and

$$p' := \frac{\mathrm{sign}(\nu_t)\nu_0 m\kappa_t}{2\tau'^2}\,\frac{[\mathrm{sign}(\nu_t)s + (|\nu_t| + \nu_0\kappa_0\kappa_t^{-1})]^{1/2}}{[-\mathrm{sign}(\nu_t)s + (|\nu_t| - \nu_0\kappa_t\kappa_0^{-1})]^{1/2}} \times (2|\nu_t|\nu_0^{-1} + \kappa_0\kappa_t^{-1} - \kappa_t\kappa_0^{-1})\,. \tag{6.14}$$

If we define

$$\nu_t' := \nu_t + \frac{1}{2}\mathrm{sign}(\nu_t)\nu_0(\kappa_0\kappa_t^{-1} - \kappa_t\kappa_0^{-1})\,, \tag{6.15}$$

we may write

$$\sqrt{-\tau'^2 + \nu_t'^2} = \mp\left[s + \frac{1}{2}\mathrm{sign}(\nu_t)\nu_0(\kappa_t\kappa_0^{-1} + \kappa_0\kappa_t^{-1})\right]. \tag{6.16}$$

The sign must be chosen so that the root is positive. Combining (6.13) and (6.16), we obtain for the energy in the moving frame (τ', t'),

$$E' = \pm m\tau'^{-1}\sqrt{-\tau'^2 + \nu_t'^2}\,. \tag{6.17}$$

E' is positive if the minus-sign holds in (6.16), and negative otherwise. The momentum we obtain as

$$p' = m\nu_t'\tau'^{-2}t'(\mathrm{sign}(E')\nu_t', \tau')\,. \tag{6.18}$$

Here $t'(\mathrm{sign}(E')\nu_t', \tau')$ is as in (5.9) with (τ, t, ν_t) replaced by $(\tau', t', \mathrm{sign}(E')\nu_t')$.

Formulas (6.13) and (6.14) (or (6.16)–(6.18)) define energy and momentum for tachyons in moving frames. A negative energy indicates time inversion, cf. Sec. 2, and a sign change during the evolution indicates a double image of a tachyon emerging in the moving frame. This we will discuss in the next section.

7. The Appearance of Tachyons in Moving Frames

We study how the trajectory of a particle/tachyon [with integration constants $(\kappa_{p,t}, \nu_{p,t})$ in the frame of absolute rest (τ, t)] appears to a moving observer [with integration constants (κ_0, ν_0) and rest frames (τ', t')]. The transformation formula relating the two coordinate frames is given in (5.11) and (5.12).

For particles this is very easy to settle. The trajectory in the frame of absolute rest is given in (5.6). The range of the curve parameter is $|\nu_p| < s < \infty$. In order that conditions (4.14) [with $z = 0$, and b, h as in (5.10)] are satisfied, we must further restrict the range of s,

$$\infty > s > s_{\min} := \max[(\nu_p - \nu_0\kappa_0\kappa_p^{-1}), (-\nu_p + \nu_0\kappa_p\kappa_0^{-1}), |\nu_p|]\,. \tag{7.1}$$

Only the part of the trajectory (5.6) parametrized in this range is mapped into the rest frame of the moving observer. Thus, the trajectory in the rest frame (τ', t') of the moving observer (κ_0, ν_0) is given by (6.3) and (6.4) with the parameter range (7.1). The trajectory (5.7) within the range $[0, \tau(s_{\min})]$ is not visible in the moving frame.

In the frame of absolute rest the trajectory of a tachyon is given in (5.8). The parameter range is $-|\nu_t| < s < 0$. The part of the trajectory which is visible in the moving frame depends on the integration parameters (κ_t, ν_t) and (κ_0, ν_0) of tachyon and observer. We will not give here a complete discussion of this parameter space, but rather pick out an example, which qualitatively comprises all possible cases. First of all, we assume $\nu_t > 0$. For conditions (4.14) are satisfied, we must have, analogous to (7.1),

$$\max[-(\nu_t + \nu_0\kappa_0\kappa_t^{-1}), -\nu_t] < s < \min[(\nu_t - \nu_0\kappa_t\kappa_0^{-1}), 0]\,. \tag{7.2}$$

Only the part of the trajectory (5.8) which is parametrized in this range is visible for the observer (κ_0, ν_0). It may well be that there is no s-value at all which satisfies this condition, and then the tachyon is never visible in the moving (τ', t')-frame. In fact, there are only three possibilities, that (7.2) has a solution.

(I) If $0 < \frac{1}{2}\nu_0\kappa_t\kappa_0^{-1} < \nu_t < \nu_0\kappa_t\kappa_0^{-1}$, then $-\nu_t < s < \nu_t - \nu_0\kappa_t\kappa_0^{-1}$.
(II) If $0 < -\nu_0\kappa_0\kappa_t^{-1} < \nu_t$, then $-(\nu_t + \nu_0\kappa_0\kappa_t^{-1}) < s < 0$.
(III) If $0 < \nu_0\kappa_t\kappa_0^{-1} < \nu_t$, then $-\nu_t < s < 0$.

Cases I–III are mutually exclusive. For $\nu_t < 0$ one gets very symmetrical conditions, which we do not list here.

The most important thing to notice is that the function $\tau'(s)$ in (6.11) need not be a monotonous function of s. In fact, it has a maximum at

$$s_{\max} = -\frac{1}{2}\nu_0(\kappa_t\kappa_0^{-1} + \kappa_0\kappa_t^{-1}), \quad \tau'(s_{\max}) = \left|\nu_t + \frac{1}{2}\nu_0(\kappa_0\kappa_t^{-1} - \kappa_t\kappa_0^{-1})\right|, \quad (7.3)$$

if $s_{\max}$ lies within the range (7.2). Note that $s_{\max}$ is just the value at which the energy (6.13) changes its sign; for $s < s_{\max}$ it is positive.

We discuss here only Case I. We have $\tau'(-\nu_t) = \sqrt{\nu_0\kappa_0\kappa_t^{-1}(2\nu_t - \nu_0\kappa_t\kappa_0^{-1})}$, and $\tau'(\nu_t - \nu_0\kappa_t\kappa_0^{-1}) = 0$. The part of the trajectory (5.8) which is parametrized in the range $\nu_t - \nu_0\kappa_t\kappa_0^{-1} < s < 0$ is not visible in the moving frame.

(I.a) If $\nu_t < \frac{1}{2}\nu_0(\kappa_0\kappa_t^{-1} + \kappa_t\kappa_0^{-1})$, then $s_{\max}$ lies outside the s-range (7.2), (i.e., the s-range indicated in Case I), and $\tau'(s)$ is a decreasing function. Since $\tau(s)$ is increasing, the time order is inverted: in the frame of absolute rest the tachyon moves from $t(-\nu_t)$ to $t(\nu_t - \nu_0\kappa_t\kappa_0^{-1})$, whereas in the rest frame of observer (κ_0, ν_0) it appears to move from $t'(\nu_t - \nu_0\kappa_t\kappa_0^{-1})$ to $t'(-\nu_t)$. The energy, E' in (6.13), is negative in the moving frame, which indicates the time inversion, cf. Sec. 2.

(I.b) If $\nu_t > \frac{1}{2}\nu_0(\kappa_0\kappa_t^{-1} + \kappa_t\kappa_0^{-1})$, then $s_{\max}$ lies within the s-range of Case I. We have $-\nu_t < s_{\max} < s_0 < \nu_t - \nu_0\kappa_t\kappa_0^{-1}$, $0 < \tau'(-\nu_t) < \tau'(s_{\max}) > \tau'(s_0) > \tau'(\nu_t - \nu_0\kappa_t\kappa_0^{-1}) = 0$, and $\tau'(s_0) = \tau'(-\nu_t)$, with $s_0 := \nu_t - \nu_0(\kappa_0\kappa_t^{-1} + \kappa_t\kappa_0^{-1})$.

We encounter here a most amazing phenomenon. The observer (κ_0, ν_0) sees at first a tachyon T_1 emerging at $\tau'(\nu_t - \nu_0\kappa_t\kappa_0^{-1}) = 0$. This tachyon moves in the time interval $[0, \tau'(s_{\max})]$, and disappears at $\tau'(s_{\max})$. However, at $\tau'(-\nu_t)$ $(= \tau'(s_0))$, when the tachyon T_1 is at $t'(s_0)$, a second tachyon T_2 appears at $t'(-\nu_t)$. T_2 exists in the range $[\tau'(-\nu_t), \tau'(s_{\max})]$ and has positive energy. Within this period the observer sees two tachyons, totally separated in space, which coalesce at $\tau'(s_{\max})$, $t'(s_{\max}) = \kappa_t$, and disappear. The part of the trajectory (5.8) in the frame of absolute rest which is parametrized in the range $s_{\max} < s < \nu_t - \nu_0\kappa_t\kappa_0^{-1}$ appears as tachyon T_1, and the part parametrized by $-\nu_t < s < s_{\max}$ as T_2. The motion of T_1 is time inverted in the moving frame; T_1 appears to move with negative energy from $t'(\nu_t - \nu_0\kappa_t\kappa_0^{-1})$ to $t'(s_{\max})$, whilst the actual motion of the tachyon in the frame of absolute rest is from $t(s_{\max})$ to $t(\nu_t - \nu_0\kappa_t\kappa_0^{-1})$. There is a finite period in which T_1 and T_2 coexist for the moving observer. In other words, a tachyon may appear at the same time at different places in the moving frame (τ', t'). After all, T_1 and T_2 correspond to one and the same tachyon in the frame of absolute rest.

8. Conclusion

I review here some aspects of the absolute cosmic spacetime conception, suggested in this article. Superluminal signals, the causality principle, and the principle of special relativity are logically contradictory; at least one of them must be dropped. The authors of Refs. 1–4 abandoned the causality principle; in this article we studied tachyons in the context of an absolute spacetime, and dropped the relativity principle.

As long as we imagine space as the void generated by co-ordinate axes, relativity principles are quite natural. A cosmological space conception, however, should be based on the galactic distribution, the galactic grid that defines cosmic space. It should be argued that this happens in general relativity, because the energy-momentum tensor of the galactic background determines to a certain extend the metric. But one can always introduce locally geodesic coordinates, and thus space remains the void generated by rectangular coordinate axes.

Our starting point is the fact that the galactic background provides a natural reference frame, a frame of absolute rest. By means of this frame the energy of tachyons can unambiguously be defined in all uniformly moving frames. In the frame of absolute rest tachyonic energy is positive. In moving frames it may become negative, but it is still bounded from below. Based on the energy concept developed in Sec. 2, we defined particle-tachyon interactions in terms of elastic head-on collisions, which are entirely determined by energy-momentum conservation, cf. Sec. 3.

In Sec. 2, it was demonstrated that the time order of events connected by superluminal signals is inverted in moving frames whenever the energy of the tachyons involved is negative there. Since a change of the time order is always accompanied by negative tachyonic energy, the observer can infer the time order in the galactic frame. In this frame, the time order is defined by cosmic time, which labels the expansion of the three-space in a RW cosmology. All observers arrive so at the same conclusion on the causality of the process they observe in their respective frames of rest.

In Sec. 7, we discussed double images of tachyons in moving frames. One and the same tachyon can simultaneously emerge at two different space points in the observer's frame of rest. Redoubling effects are likewise indicated to the moving observer by negative tachyonic energy.

The existence of superluminal signals is not in disagreement with standard relativity theory, though this is often claimed. Einstein's relativity exclusively deals with (sub)luminal particles, and no conclusions about the existence, or, for that matter, non-existence of superluminal particles can be derived from this theory. The theory of superluminal motion developed here does not imply any modifications of standard relativity theory for subluminal particles either. If events are connected by (sub)luminal signals, their spacetime relation is qualitatively the same for all geodesically moving observers.

If, however, superluminal particles are included in a physical process, then observations of observers in relative motion may qualitatively differ. In this case, one cannot assume a relativity principle, i.e., regard observations in all uniformly moving frames as equally real. The galactic background defines a state of absolute rest. Only observers in this state are able to perceive reality, whereas observers moving in the galactic grid may well see a completely illusory causality. However, moving observers can infer from the galactic background passing by the real causal connections and the proper cosmic time order in the frame of absolute rest.

This cosmological approach to superluminal motion has two distinctive advantages compared to the standard theory.[1,4] Causality is preserved, and tachyon-particle interactions can be defined completely classically, without resorting to the quantum mechanical antiparticle concept.

In this paper we investigated hypothetical particles whose speed always exceeds that of light, in the galactic frame of absolute rest as well as in the rest frames of uniformly moving observers. In Refs. 9 and 10, a different way of generating superluminal motion is explored, a RW cosmology in which the dynamics of particles and rays is determined by a permeability tensor representing the substance of space — the cosmic ether.[11] The meaning of privileged coordinate frames in the context of general relativity is discussed in Ref. 12, although from a completely different viewpoint. Finally, Refs. 13 and 14 give an account on cosmic chaos and its implications on the absolute spacetime conception advocated here.

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