Class. Quantum Grav. **16** (1999) 3349–3366. Printed in the UK PII: S0264-9381(99)02789-6

Tachyons in the Milne universe

Roman Tomaschitz

Department of Physics, Hiroshima University, 1-3-1 Kagami-yama, Higashi-Hiroshima 739-8526, Japan

E-mail: roman@fusion.sci.hiroshima-u.ac.jp

Received 19 March 1999, in final form 9 August 1999

Abstract. Superluminal particles (tachyons) are studied in a Robertson–Walker cosmology with linear expansion factor and negatively curved 3-space (Milne universe). This cosmology admits globally geodesic rest frames for uniformly moving observers, isometric copies of the forward lightcone, which can be synchronized by Lorentz boosts. We investigate superluminal wave propagation, a real Proca field with negative mass-square, coupled to subluminal matter in analogy to the electromagnetic field. For photons, the eikonal approximation is exact in Robertson–Walker cosmology, and the Proca field is coupled to the background geometry in such a way that this also holds for tachyons. The spectral decomposition of freely propagating tachyon fields in the Milne universe is derived. We study the wave–particle duality in terms of the spectral elementary waves and their orthogonal ray bundles, in the comoving frame as well as in the individual geodesic rest frames of galactic observers. The spectral energy density of a tachyon background radiation is discussed.

PACS numbers: 9880H, 9870V, 0350K, 0365S

1. Introduction

The possibility of superluminal radiation is investigated in a Robertson–Walker (RW) cosmology with negatively curved 3-space and a linear expansion factor (Milne universe [1–5]). This cosmology deserves special attention because it is isometric to the forward lightcone, so that globally geodesic rest frames can be introduced for geodesically moving observers [5–8], which admit a well defined global synchronization by Lorentz boosts. Although a flat 4-manifold, this universe is expanding, and gives rise to genuine redshifts, which emerge in geodesic rest frames as ordinary relativistic Doppler shifts. In contrast, in a general RW cosmology, and even in de Sitter cosmologies, these properties hold only locally and approximately in locally geodesic neighbourhoods. Though the Milne universe has long been discussed in the context of observational cosmology [4, 5, 9], it has never gained great popularity due to the fact that it is based on a flat spacetime; thus the curvature tensor vanishes and hence, via the Einstein equations, the energy–momentum tensor. However, the possibilities of evolution of an open universe go far beyond what is predictable by Einstein's equations [10], and there does not seem to be a particularly satisfactory solution for the expansion factor, whatever values one chooses for the Hubble constant, deceleration parameter and the sign of the space curvature [11].

Modern theories of superluminal motion [12–14] are based on the formalism of classical relativistic mechanics. Faster-than-light particles (tachyons [12]) are usually introduced as an extension of the relativistic particle concept, as particles with negative mass-square, or, if one prefers, imaginary mass. In the relativistic Hamilton–Jacobi equation, this just means assuming $m^2 < 0$ without further alterations. If tachyons are assumed to carry electric charge, then their coupling to the electromagnetic potential is commonly effected by minimal substitution, and the Lagrangian for tachyons can be written in the same way as for subluminal particles, $L = -\sqrt{-m^2\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu} + eA_\mu\dot{x}^\mu$, but with $m^2 < 0$. (diag$(\eta_{\mu\nu}) = (-1, 1, 1, 1)$; $\eta_{\mu\nu}$ may be replaced by a Riemannian metric, of course.) Sommerfeld's pre-relativistic study of superluminal motion [15] aimed at accelerating electrons beyond the speed of light by means of electromagnetic fields, but otherwise his view of tachyons as point particles coupled in the usual way to the electromagnetic field was taken over by modern authors.

In this paper, a different approach to superluminal motion is investigated. Superluminal wave propagation is modelled in analogy to classical electrodynamics, by a Proca equation with negative mass-square, very much in contrast to the prevailing view of tachyons as charged point particles with imaginary mass and zero spin. The superluminal wave modes of the tachyon field are coupled to a current of subluminal massive particles by minimal substitution. In short, in the above Lagrangian we regard the field as superluminal rather than the particle.

We study tachyonic wave propagation in the comoving RW frame of the Milne universe as well as in the individual, globally geodesic rest frames of galactic observers, which are copies of the forward lightcone. (A recent review on the synchronization of coordinate frames in relativity theory can be found in [16].) The tachyon field is conformally coupled to the background metric, therefore the semiclassical approximation is exact for freely propagating tachyons in RW cosmology, the classical action coincides with the phase of the spectral elementary waves. On that basis we partition the classical rays into bundles labelled by the wavefronts of the spectral waves, and define in this way the energy and frequency of tachyons along individual rays, thus arriving at a very elementary geometric description of the wave–particle dualism.

In section 2 the classical mechanics of tachyons is discussed, in particular the energy concept for tachyons in the comoving frame. We introduce a scalar wave equation for superluminal particles, designed in a way that the semiclassical approximation is exact [17, 18]. Then this real scalar field is extended to a vector field so that the phase of the spectral waves remains unchanged and the wave propagation is transversal. We arrive in this way at a real Proca equation with negative mass-square.

In section 3 we discuss how tachyons appear in the geodesic rest frames of galactic observers. The mode decomposition of the tachyon field, the relation of spectral modes to tachyonic worldlines, and the tachyon energy are derived in the lightcone representation of the Milne universe. In section 4 we present our conclusions, and discuss the spectral energy density of tachyon background radiation. In appendix A we sketch the action of the Lorentz group in the comoving frame, and map it onto the standard representation in the globally geodesic rest frames. In appendix B we construct a complete set of spectral waves for the electromagnetic field in the forward lightcone, and discuss the geometry of wavefronts and the eikonal approximation. In this paper we focus on semiclassical mechanics; studies on second quantization in the Milne universe can be found in [19–23].

2. The wave equation for tachyons and the semiclassical approximation

We consider a RW cosmology defined by the line element

$$ds^2 = -d\tau^2 + a^2(\tau)\, d\sigma^2, \tag{2.1}$$

and use as a coordinate representation of the open hyperbolic 3-space the Poincaré half-space H^3 with the line element [24, 25]

$$\mathrm{d}\sigma^2 = u^{-2}(\mathrm{d}u^2 + |\mathrm{d}\xi|^2), \tag{2.2}$$

with Cartesian coordinates u, ξ_1, ξ_2; $u > 0$ and $\xi = \xi_1 + \mathrm{i}\xi_2$. $\mathrm{d}\sigma^2$ induces constant negative curvature -1 on this half-space. If the expansion is linear, $a(\tau) = \tau$, $\tau > 0$, then this spacetime is isometric to the (interior of the) forward lightcone, $t^2 - |\boldsymbol{x}|^2 > 0$, $t > 0$, endowed with the Minkowski metric $\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}\boldsymbol{x}^2$ (Milne universe [1]), and the isometry reads [18]

$$\begin{pmatrix} t \\ x \end{pmatrix} = \frac{\tau}{2u}(\xi_1^2 + \xi_2^2 + u^2 \pm 1), \qquad (y, z) = \frac{\tau}{u}(\xi_1, \xi_2), \tag{2.3}$$

$$\tau = \sqrt{t^2 - |\boldsymbol{x}|^2}, \qquad u = \frac{1}{t - x}\sqrt{t^2 - |\boldsymbol{x}|^2}, \qquad (\xi_1, \xi_2) = \frac{1}{t - x}(y, z). \tag{2.4}$$

In appendix A, we give an explicit representation of the symmetry group of the Milne universe in the comoving RW frame (2.1), and show how it relates to the standard representation of the proper orthochronous Lorentz group in the lightcone.

Tachyonic worldlines are defined by a Hamilton–Jacobi equation with negative mass-square, $g^{\mu\nu}S_{,\mu}S_{,\nu} = \mu^2$. ($\mu^2 > 0$ in our notation, with $g_{\mu\nu}$ as given by (2.1).) This is equivalent to the action $S = \int L\,\mathrm{d}\lambda$, $L(\lambda) = -\mu\sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}$. At first we study worldlines along the u-semiaxis of H^3 (or along straight lines, ξ = constant, orthogonal to the complex plane). We have two integrals of motion,

$$\mathrm{d}\log u = s a^{-1}(s^2 - a^2\mu^2)^{-1/2}\,\mathrm{d}\tau, \qquad \mathrm{d}\lambda = a(s^2 - a^2\mu^2)^{-1/2}\,\mathrm{d}\tau; \tag{2.5}$$

s is an integration constant. We assume that the tachyon mass is a scalar varying in cosmic time inversely proportional to the curvature radius of the 3-space, $\mu = m/a(\tau)$, $m > 0$. (The wave equation for tachyons is conformally coupled, which requires this scaling of the tachyon mass, see the discussion following equation (2.9). A possible cosmic time variation of the rest mass of subluminal particles is discussed in [2].) The worldlines along the u-semiaxis read

$$u(\tau) = u_0 \exp\left(\delta(s)\int_{\tau_0}^{\tau} a^{-1}\,\mathrm{d}\tau\right), \qquad \delta(s) := \frac{s}{\sqrt{s^2 - m^2}}, \tag{2.6}$$

with $|s| > m$. In the following we consider linear expansion, $a(\tau) = \tau$. If we focus on trajectories orthogonal to the complex plane, we can use the action

$$S = -\sqrt{s^2 - m^2}\log(\tau/\tau_0) + s\log(u/u_0), \tag{2.7}$$

since the trajectories (2.6), namely $\tau^\delta u$ = constant, are recovered from $\partial S/\partial s = 0$. Tachyonic energy and momentum along the u-semiaxis read as

$$E = \frac{\mu}{\sqrt{|\boldsymbol{v}|^2 - 1}} = \frac{\mathrm{d}\tau}{\mathrm{d}\lambda} = \frac{1}{\tau}\sqrt{s^2 - m^2}, \qquad |\boldsymbol{v}| = |\delta(s)| \tag{2.8}$$

$$\boldsymbol{p} = \frac{\mu\boldsymbol{v}}{\sqrt{|\boldsymbol{v}|^2 - 1}} = \frac{\mathrm{d}u}{\mathrm{d}\lambda} = \frac{su}{\tau^2}, \qquad |\boldsymbol{p}| = \frac{|s|}{\tau}. \tag{2.9}$$

In the case of photons, the eikonal approximation is exact in RW cosmologies, irrespective of the expansion factor, cf appendix B. We also want to retain this property for particles with negative mass-square. We consider a real scalar field,

$$L = -\tfrac{1}{2}[\psi_{,\nu}\psi^{,\nu} + (\tfrac{1}{6}R - \mu^2)\psi^2], \qquad \tfrac{1}{6}R := -\frac{1}{a^2} + \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}. \tag{2.10}$$

The insertion of the Ricci scalar and the scaling of the mass with the inverse of the expansion factor as indicated after (2.5) guarantees the conformal coupling of the wave equation,

$$\psi_{,\mu}{}^{;\mu} - \left(\tfrac{1}{6}R - \mu^2\right)\psi = 0, \tag{2.11}$$

so that the frequencies of the eigenmodes scale with the inverse of the expansion factor. We consider wave fields independent of the ξ-coordinate (cf equation (B.1)). The separation ansatz $\psi(\tau) = \chi(\tau)\, a^{-3/2} u^{1-\mathrm{i}s}$ gives

$$\frac{\mathrm{d}^2\chi}{\mathrm{d}\tau^2} + \left(\frac{1}{a^2}s^2 - \mu^2 - \frac{1}{2}\frac{\ddot{a}}{a} + \frac{1}{4}\frac{\dot{a}^2}{a^2}\right)\chi = 0. \tag{2.12}$$

As $\mu = m/a(\tau)$, we have the fundamental solutions

$$\chi = \sqrt{a}\exp\left(\mp\mathrm{i}\sqrt{s^2 - m^2}\int a^{-1}\,\mathrm{d}\tau\right). \tag{2.13}$$

For $a(\tau) = \tau$, we obtain as solutions of the wave equation (2.11)

$$\psi(\tau, u; s) = \tau^{-1} u \exp(-\mathrm{i}S), \tag{2.14}$$

(real and imaginary parts, we use in the following complex notation for convenience as in appendix B), with the action S given in (2.7). The energy of the wave fields (2.14) is defined as $E(s) = -\partial S/\partial t$, which coincides with the energy (2.8), of course.

Remarks. To restore the natural units $c, \hbar$ and $R_0 = c/H_0 = c\tau_0$ (present-day curvature radius) in the wave equation (2.11), we replace m by $mc/\hbar$, and in formula (2.10) for the Ricci scalar R we multiply the first term by R_0^{-2}, and the second and third terms by c^{-2}. The curvature radius of the 3-space reads as $R_0 a(\tau)$, $a(\tau) = \tau/\tau_0$. The frequency and wavevector are related to energy and momentum as $\omega = E/\hbar$ and $\boldsymbol{k} = \boldsymbol{p}/\hbar$, with $\boldsymbol{p} = g^{uu}\partial S/\partial u$, cf equation (2.9), because the semiclassical approximation is exact. (For the rest of this section, ω denotes a complex spectral variable.) For the same reason we can identify the photon energy with frequency, cf appendix B. We obtain for the group and phase velocity $|\boldsymbol{v}_{\mathrm{gr}}| = c^2|\boldsymbol{v}_{\mathrm{ph}}|^{-1} = c|\delta(s)|$. The group and particle velocity coincide, and can be made arbitrarily large by the choice of the integration parameter (spectral variable) s. By means of the comoving galaxy frame, the energy of tachyons can be defined unambiguously in all geodesic rest frames, without resorting to the quantum mechanical antiparticle concept [12], and the causality principle is preserved, as the cosmic time of the comoving frame determines a distinguished time order to which every observer can relate [6, 26].

A complete set of elementary waves $\psi(\tau, u, \xi; s, \omega)$ in H^3 is generated by applying the symmetry transformations (B.3) to the scalar wave fields (2.14). This just means substituting into (2.14) the Poisson kernel (B.6) for u; in particular,

$$S(\tau, u, \xi; s, \omega) = s\log[(\tau_0/\tau)^{1/\delta} P(u, \xi; \omega)/u_0] \tag{2.15}$$

is the general solution of the Hamilton–Jacobi equation as well as the phase of the spectral elementary waves.

A ray bundle of tachyonic worldlines is associated to every spectral wave as follows. We consider the horospheres $\partial S/\partial s = 0$, which explicitly read as

$$\left(u - \frac{1}{2u_0}(\tau_0/\tau)^{\delta}\right)^2 + |\xi - \omega|^2 = \frac{1}{4u_0^2}(\tau_0/\tau)^{2\delta}, \tag{2.16}$$

cf equation (B.13). If $s > m$, these Euclidean spheres are contracting (expanding) for $\tau \to \infty$ ($\tau \to 0$) and expanding (contracting) if $s < -m$. The focal point of the ray bundle orthogonal to these spheres is the point ω on the boundary of H^3. Applying the transformation (A.1), (B.15) onto $(u(\tau) = \kappa_0(\tau_0/\tau)^\delta, \xi = 0)$, we obtain

$$u(\tau) = \frac{2\kappa\kappa_0(\tau_0/\tau)^\delta}{1 + \kappa_0^2(\tau_0/\tau)^{2\delta}}, \qquad \xi(\tau) = \omega + \frac{2\kappa\kappa_0^2(\tau_0/\tau)^{2\delta}\exp(\mathrm{i}\varphi)}{1 + \kappa_0^2(\tau_0/\tau)^{2\delta}}. \tag{2.17}$$

The rays of this bundle terminate in ω, if $\delta(s) > 1$, or emanate from ω, if $\delta(s) < -1$; the velocity along the rays is $|\boldsymbol{v}| = |\delta(s)|$. The bundle (2.17) is evidently the tachyonic counterpart to (B.16)–(B.18). (Positive and negative frequencies are distinguished by the sign of δ.) The rays (2.17) satisfy identically $P(u, \xi; \omega) = (2\kappa\kappa_0)^{-1}(\tau/\tau_0)^\delta$. Thus, if we choose $\kappa = 1/(2\kappa_0 u_0)$ in (2.17), then this bundle solves (2.16). The trajectory $(u = u_0^{-1}(\tau_0/\tau)^\delta, \xi = \omega)$, which likewise solves (2.16), completes the bundle. The velocity and wavevector along the rays (2.17) are related by

$$\begin{aligned} \boldsymbol{k}_i &= \partial S/\partial(u, \xi) = s\kappa^{-1}[\sinh(\log(\kappa_0(\tau_0/\tau)^\delta)), -\exp(\mathrm{i}\varphi)], \\ \boldsymbol{k}^i &= \tau^{-1}\sqrt{s^2 - m^2}\, \boldsymbol{v}^i, \qquad \boldsymbol{v}^i = (\mathrm{d}u/\mathrm{d}\tau, \mathrm{d}\xi/\mathrm{d}\tau), \end{aligned} \tag{2.18}$$

which is the tachyonic version of (B.20).

The vectorial extension of the wave equation (2.11) is based on the Lagrangian

$$L = -\tfrac{1}{4}F_{\alpha\beta}F^{\alpha\beta} + \tfrac{1}{2}\mu^2 A_\alpha A^\alpha + A_\alpha j^\alpha, \tag{2.19}$$

with $F_{\alpha\beta} = A_{\beta,\alpha} - A_{\alpha,\beta}$ and $\mu = m/a(\tau)$, $m > 0$, which leads to the Proca equation [27, 28]

$$\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}F^{\alpha\beta})}{\partial x^\beta} - \mu^2 A^\alpha = j^\alpha, \tag{2.20}$$

with negative mass-square for a real vector potential A_α. Differentiating (2.20), and using the skew symmetry of $F^{\alpha\beta}$, we obtain

$$A^\alpha{}_{;\alpha} + \frac{\mu^2{}_{,\alpha}}{\mu^2}A^\alpha = -\frac{1}{\mu^2}j^\alpha{}_{;\alpha}. \tag{2.21}$$

If the current is conserved, this is the analogue to the Lorentz condition, but unlike the Lorentz condition, equation (2.21) is a consequence of the field equations, and the mass term also breaks the gauge symmetry.

Remarks. The tachyon mass μ scales inversely proportional with the expansion factor, as in (2.11), in order to retain the conformal coupling of the Maxwell field; also note that μ is not a rest mass, and Planck's constant enters into this classical field theory to give the mass term the right dimension in the Lagrangian, $\mu = mc/(\hbar a(\tau))$ (with $a(\tau_0) = 1$ at the present epoch). The cosmic time scaling of the tachyon mass clearly reminds one of the varying fundamental constants of Eddington, Milne and Dirac [2, 8, 29, 30]. The Lagrangian (2.19) suggests a well defined interaction of tachyon fields with subluminal matter, quite analogous to classical electrodynamics; the current in (2.20) is assumed to be structured as in electrodynamics. The choice of the Lagrangian (2.19) as an extension of classical electrodynamics rather than of subluminal massive quantum field theories [12, 13] is also motivated by the fact that tachyons do not have a rest mass. The tachyon field can be manifest by cosmic background radiation as well as in atomic energy levels, cf section 4 for further discussions.

To solve the wave equation (2.20) with $j^\alpha = 0$, i.e. to find its spectral resolution, we try the separation ansatz $A_0 = A_1 = A_3 = 0$, $A_2 = \varphi(\tau)\, u^{\mathrm{i}s}$, so that $F^{02} = -a^{-2}\varphi'(\tau)\, u^{2+\mathrm{i}s}$ and $F^{21} = -\mathrm{i}sa^{-4}\varphi(\tau)\, u^{3+\mathrm{i}s}$, all other components of $F^{\alpha\beta}$ vanish. In this way the $\alpha = 0, 1, 3$ components of the field equation (2.20) are identically satisfied, and the $\alpha = 2$ component leads, if we write $\varphi = a^{-1/2}\chi$, just to equation (2.12). Accordingly, the phase of the spectral modes coincides with that of the scalar wave fields (2.14). A second independent set of transversal modes is obtained by interchanging A_2 and A_3 in the above ansatz. Finally, a complete set of eigenmodes for the wave equation (2.20) is generated by applying the symmetry transformations (B.3) to these two linearly polarized sets. The phase of the spectral modes is the action (2.15).

3. Tachyons in the forward lightcone

The mode decomposition of the free wave equation (2.20) for tachyons (with $\mu = m/\tau$) is readily obtained from appendix B, if we replace in (B.1) and (B.4) the factor $\tau^{\mp \mathrm{i}s}$ by $\tau^{\mp \mathrm{i}\sqrt{s^2-m^2}}$. In the forward lightcone, this means multiplying the wave fields (B.8)–(B.11) by $(t^2 - |\boldsymbol{x}|^2)^{\mp \mathrm{i}(\sqrt{s^2-m^2}-s)/2}$. All the procedures outlined below, relating bundles of tachyonic worldlines to the spectral waves, are quite analogous to the discussion of the electromagnetic field in appendix B.

We obtain the action in the forward lightcone by inserting (2.4) into (2.15),

$$S(t, \boldsymbol{x}; s, \omega_1, \omega_2) = s \log[(t^2 - |\boldsymbol{x}|^2)^{-1/(2\delta)} P(t, \boldsymbol{x}; \omega_1, \omega_2)], \tag{3.1}$$

with $P(t, \boldsymbol{x}; \omega_1, \omega_2)$ as in (B.22). The phase corresponding to the action (2.7) reads in the lightcone as

$$S(t, \boldsymbol{x}; s) = s \log[(t^2 - |\boldsymbol{x}|^2)^{1/2-1/(2\delta)}(t - x)^{-1}]. \tag{3.2}$$

The ray bundle (2.17) is mapped by (2.3) into the lightcone,

$$\begin{gathered}\begin{pmatrix} t \\ x \end{pmatrix} = \frac{\tau}{4\kappa}[(|\omega|^2 \pm 1)(\alpha\tau)^{\delta} + (|\omega|^2 \pm 1 + 4\kappa^2 + 4\kappa \operatorname{Re}(\omega\, \mathrm{e}^{-\mathrm{i}\varphi}))(\alpha\tau)^{-\delta}], \\ y + \mathrm{i}z = \frac{\tau}{2\kappa}[\omega(\alpha\tau)^{\delta} + (\omega + 2\kappa \mathrm{e}^{\mathrm{i}\varphi})(\alpha\tau)^{-\delta}], \qquad (t + \boldsymbol{n}\boldsymbol{x}) = \frac{2\kappa\alpha^{-\delta}}{1 + |\omega|^2}\tau^{1-\delta}.\end{gathered} \tag{3.3}$$

(α is positive and related to τ_0 and κ_0 in (2.17) by $(\alpha\tau_0)^{-\delta} = \kappa_0$, and $\boldsymbol{n}$ is defined in (B.22).) The trajectories are parametrized by cosmic time and satisfy $\partial S/\partial s =$ constant, or

$$2\kappa(t^2 - |\boldsymbol{x}|^2)^{(1-\delta)/2} = \alpha^{\delta}(1 + |\omega|^2)(t + \boldsymbol{n}\boldsymbol{x}); \tag{3.4}$$

where $\boldsymbol{n}$ is the symmetry axis of these surfaces, which are deformations (parametrized by δ) of the horospheres/planes studied in appendix B.

The time coordinate $t(\tau)$ in (3.3) diverges for $\tau \to 0$ as well as for $\tau \to \infty$, and it attains its minimum at

$$\begin{gathered}\tau_{\min} = \frac{1}{\alpha}\left(\frac{(\delta - 1)[|\omega|^2 + 1 + 4\kappa^2 + 4\kappa \operatorname{Re}(\omega\, \mathrm{e}^{-\mathrm{i}\varphi})]}{(\delta + 1)(|\omega^2| + 1)}\right)^{1/(2\delta)}, \\ t(\tau_{\min}) = \frac{\tau_{\min}|\delta|\sqrt{|\omega|^2 + 1}}{2\kappa\sqrt{\delta^2 - 1}}[|\omega|^2 + 1 + 4\kappa^2 + 4\kappa \operatorname{Re}(\omega\, \mathrm{e}^{-\mathrm{i}\varphi})]^{1/2}.\end{gathered} \tag{3.5}$$

The worldlines orthogonal to the complex plane ($u = (\alpha\tau)^{\delta}, \xi = \omega$), cf equation (2.6), which have the point at infinity as the focal point, are mapped onto

$$\begin{pmatrix} t \\ x \end{pmatrix} = \tfrac{1}{2}\tau[(\alpha\tau)^{\delta} + (|\omega|^2 \pm 1)(\alpha\tau)^{-\delta}], \qquad y + \mathrm{i}z = \tau\omega(\alpha\tau)^{-\delta}, \tag{3.6}$$

and satisfy

$$(t^2 - |\boldsymbol{x}|^2)^{(1-\delta)/2} = \alpha^\delta (t - x). \tag{3.7}$$

The minimum of $t(\tau)$ now occurs at

$$\tau_{\min} = \frac{1}{\alpha}\left(\frac{(\delta-1)(|\omega|^2+1)}{\delta+1}\right)^{1/(2\delta)}, \qquad t(\tau_{\min}) = \frac{\tau_{\min}|\delta|\sqrt{|\omega|^2+1}}{\sqrt{\delta^2-1}}. \tag{3.8}$$

Accordingly, a double image of the tachyon appears in the geodesic rest frame in the time interval $[t(\tau_{\min}), \infty]$, whereas the tachyon is not visible at all in $[0, t(\tau_{\min})]$; this is further discussed after equation (3.13).

The wave equation (2.11) reads in the forward lightcone as $\psi_{,\mu}{}^{,\mu} + \mu^2\psi = 0$, with $\mu = m/\sqrt{t^2 - |\boldsymbol{x}|^2}$. Corresponding to the action S in (3.1), we find a set of eigenmodes $\psi = (t + \boldsymbol{nx})^{-1}\exp(-\mathrm{i}S)$, cf equations (2.14) and (2.15), with energy

$$\begin{aligned}
E &= -S_{,t} = \frac{s}{4\kappa\delta\tau}U^+(\omega,\varphi,\kappa,\alpha,\delta) = \frac{\mu\,\mathrm{sign}(U^+)}{\sqrt{\boldsymbol{v}^2-1}},\\
S_{,t} &= -\frac{s}{\delta}\frac{\delta(t^2-|\boldsymbol{x}|^2) + (1-\delta)t(t+\boldsymbol{nx})}{(t+\boldsymbol{nx})(t^2-|\boldsymbol{x}|^2)},\\
U^{\pm}(\omega,\varphi,\kappa,\alpha,\delta) &:= (|\omega|^2 \pm 1)(1+\delta)(\alpha\tau)^\delta\\
&\quad + (|\omega|^2 \pm 1 + 4\kappa^2 + 4\kappa\,\mathrm{Re}(\omega \mathrm{e}^{-\mathrm{i}\varphi}))(1-\delta)(\alpha\tau)^{-\delta},
\end{aligned} \tag{3.9}$$

and $\delta(s)$ as in (2.6). In contrast to photons, cf appendix B, the energy of tachyons is no longer constant along a given ray. The particle (or group) velocity $\boldsymbol{v}$ along the rays of the bundle (3.3) is given by

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{U^-}{U^+}, \qquad \frac{\mathrm{d}(y+\mathrm{i}z)}{\mathrm{d}t} = \frac{2}{U^+}[\omega(1+\delta)(\alpha\tau)^\delta + (\omega + 2\kappa \mathrm{e}^{\mathrm{i}\varphi})(1-\delta)(\alpha\tau)^{-\delta}]. \tag{3.10}$$

The spectral elementary waves with phase (3.2), $\psi = (t-x)^{-1}\exp(-\mathrm{i}S)$, carry the energy

$$\begin{aligned}
E &= -S_{,t} = \frac{s}{2\delta\tau}V^+ = \frac{\mu\,\mathrm{sign}(V^+)}{\sqrt{\boldsymbol{v}^2-1}},\\
S_{,t} &= -\frac{s}{\delta}\frac{\delta(t^2-|\boldsymbol{x}|^2) + (1-\delta)t(t-x)}{(t-x)(t^2-|\boldsymbol{x}|^2)},\\
V^{\pm}(\omega,\alpha,\delta) &:= (1+\delta)(\alpha\tau)^\delta + (|\omega|^2 \pm 1)(1-\delta)(\alpha\tau)^{-\delta}.
\end{aligned} \tag{3.11}$$

The tachyon velocity $\boldsymbol{v}$ along the rays of the bundle (3.6) reads

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{V^-}{V^+}, \qquad \frac{\mathrm{d}(y+\mathrm{i}z)}{\mathrm{d}t} = \frac{2}{V^+}\omega(1-\delta)(\alpha\tau)^{-\delta}. \tag{3.12}$$

Note that in both cases, equations (3.9) and (3.11),

$$E(\tau\to 0) \to -\infty, \qquad E(\tau\to\infty)\to\infty, \qquad E(\tau_{\min}) = 0, \tag{3.13}$$

cf equations (3.5) and (3.8). Negative energy in individual rest frames always indicates time inversion, which can lead to double images. Let us consider a tachyon in the comoving frame, heading along the u-semiaxis from space point A to C via $B = (\tau_{\min}, u(\tau_{\min}))$, so that $\tau_A < \tau_{\min} < \tau_C$. In the geodesic rest frame, two tachyons appear simultaneously at $B = (t(\tau_{\min}), x(\tau_{\min}))$ moving in opposite directions, one towards A with negative energy and the other with positive energy toward C. The tachyon with negative energy causes the illusion of causality violation in the geodesic rest frame; the emission of the tachyon at B appears as the cause of its absorption at A, since the observer assumes that the cause precedes the effect, see [26] for a more illustrative example. In actual fact there is only one tachyon, moving from A to B and further on to C in cosmic time, and every observer can realize this by comparing the proper time of his individual geodesic rest frame to the cosmic time of the comoving galaxy frame [8].

4. Conclusion

In the cosmology considered here, one can attach to every uniformly moving observer a globally geodesic rest frame, a copy of the forward lightcone. These individual rest frames of geodesic observers are related by Lorentz transformations. The common rest frame of galactic observers, i.e. the comoving frame in which all galaxies have constant space coordinates, defines a universal cosmic time. Every geodesic observer can figure out, from the galaxy background passing by, the coordinate transformation relating his individual rest frame to the comoving galaxy frame, and can in this way synchronize his proper time with the universal cosmic time. The high isotropy of the microwave background makes it in practice possible for every observer to determine his movement relative to the galaxy background, and to infer the cosmic time order of events connected by tachyons. The distinctive feature of this cosmology is that the synchronization of clocks can be carried out in a straightforward way, without resorting to chains of infinitesimal, locally geodesic neighbourhoods, but otherwise most of the reasoning in this paper is not bound to a specific expansion factor, given that the fields considered are conformally coupled to the background geometry.

Photon frequencies scale with the inverse of the curvature radius of the 3-space; this scaling can be retained for tachyon frequencies, provided the mass of the tachyon field scales in the same way with cosmic time. The classical action then coincides with the phase of the spectral elementary waves, and tachyon frequencies and wavevectors relate to energy and momentum by the Einstein/de Broglie relation, cf section 2.

As the tachyon frequencies depend on cosmic time only via a scale factor, one can use the equilibrium distribution of a free Bose gas to describe a tachyon background radiation, and scale the time dependence of the frequencies into the temperature variable [31]. Based on the free wave equation (2.20), we obtain for the spectral energy density

$$\rho(\nu)\,\mathrm{d}\nu = \frac{8\pi}{h^3}\frac{E(p)\,p^2\,\mathrm{d}p}{\exp[E(p)/(kT(\tau))]-1} = \frac{8\pi h}{c^3}\frac{\mathrm{d}\nu\,\nu^2\sqrt{\nu^2+m^2c^4/h^2}}{\exp[h\nu/(kT(\tau))]-1}, \tag{4.1}$$

$$E(p) = h\nu = c\sqrt{p^2-(mc)^2}, \qquad T(\tau) = T_0/a(\tau).$$

There is no chemical potential in the distribution (4.1). Tachyons, like photons, are not interacting with each other. Thus, for equilibrium to be reached, we must assume interaction with subluminal matter, absorption and emission processes. Therefore, N cannot be kept constant, and then $\partial F(T,V,N)/\partial N = \mu = 0$ is a necessary extremal condition for equilibrium. The spectral energy density (4.1) can be obtained by box quantization. To this end, we use as the coordinate representation of the 3-space the ball model of hyperbolic geometry, defined by the line element $\mathrm{d}\sigma^2 = 4(1-|\boldsymbol{x}|^2/R_0^2)^{-2}\,\mathrm{d}\boldsymbol{x}^2$, $|\boldsymbol{x}| < R_0$, isometric to (2.2), cf [24, 25]. (In equation (2.2) we have put $R_0 = 1$.) The Poisson kernel (B.6), (B.4), from which the wavevectors can be extracted, reads in these coordinates [32] as

$$P^{\mathrm{i}s}(\boldsymbol{x},\boldsymbol{\eta}) = \left(\frac{1-|\boldsymbol{x}|^2/R_0^2}{|\boldsymbol{x}/R_0-\boldsymbol{\eta}|^2}\right)^{\mathrm{i}s} = \exp(-2\mathrm{i}s\boldsymbol{\eta}\boldsymbol{x}/R_0)(1+\mathrm{O}(s|\boldsymbol{x}|^2/R_0^2)). \tag{4.2}$$

We have $s = R_0/\lambda\!\!\!^{-}_0$, $\boldsymbol{k} = \boldsymbol{k}_0 a^{-1}(\tau)$, $\boldsymbol{k}_0 = \boldsymbol{\eta}s/R_0$, see the remarks following (2.14); the spectral parameter $\boldsymbol{\eta}$ is a Euclidean unit vector. The factor of two in the exponential in (4.2) arises because $d\sigma^2 \sim 4\,d\boldsymbol{x}^2$, if $|\boldsymbol{x}| \ll R_0$. For the O-term to be small within a Euclidean box of size L, we have to require $L^2 \ll R_0 ƛ_0$. Then we may apply standard Euclidean box quantization, and, if $ƛ_0 \ll L$, replace the lattice sum for the partition function over the discrete spectrum by the continuum limit, thus arriving at the spectral energy density (4.1). The line of reasoning here with regard to box quantization is somewhat different from a closed, positively curved universe, where the 3-space itself serves as a box to obtain the density of the eigenmodes [33]. To see whether these inequalities are satisfied, we have to figure out the relevant wavelengths of the tachyon radiation. As $p \geqslant mc$ in (4.1), this means $ƛ_0 < \hbar/(mc)$. We may take the order of magnitude of the present curvature radius as $R_0 \approx c/H_0 \approx 1.3 \times 10^{28}$ cm, which certainly holds for a Milne universe. Before we can proceed further, we need an estimate on the tachyon mass.

The tachyon potential of a static point source can readily be calculated from the field equations (2.20). We consider the local Euclidean limit, and regard it as a perturbation of the Coulomb potential in the hydrogen atom or a hydrogen-like ion,

$$U(r) = \frac{-\alpha}{r} + \frac{\beta}{r}\sin\left(\frac{mc}{\hbar}r\right). \tag{4.3}$$

The integration constant in the tachyon potential is fixed by requiring a finite tachyonic self-energy, so that the singularity in (4.3) entirely stems from the Coulomb potential. We can study the effect of the tachyon potential on energy levels by Bohr–Sommerfeld quantization, and an estimate on the tachyon mass is obtained by comparing high-precision measurements of the 1S–$2S_{1/2}$ and Ly-α_1, transitions in hydrogen with Lamb shift calculations [34]. We find a lower bound of 3.06 keV$/c^2$ for the tachyon mass, corresponding to a Compton wavelength of $ƛ_0^{\mathrm{C}} \approx 6.45 \times 10^{-9}$ cm. (An estimate for the coupling constant of the tachyon potential is likewise obtained in this way; we find, with the indicated tachyon mass, $\beta/\alpha \approx 9.3 \times 10^{-12}$ for hydrogen.) As the tachyon radiation is in equilibrium with the photon background, we have $mc^2/(kT_0) \approx 1.3 \times 10^7$; low-temperature expansions of the thermodynamic variables are discussed in [35]. $\rho(\nu)$ exponentially decays for $h\nu > kT_0$, i.e. for $\nu > 5.7 \times 10^{10}$ s^{-1}, and frequency and wavelength relate as $1/\lambda = \sqrt{\nu^2/c^2 + m^2c^2/h^2}$. Accordingly, $ƛ_0$ is still very close to $ƛ_0^{\mathrm{C}}$ when the exponential decay sets in, and the above inequality $L^2 \ll R_0ƛ_0$ thus holds for a very large box size, so that the continuum limit (4.1) is by all standards realized in the relevant frequency range, as well as in the look-back time, by virtue of the conformal $a(\tau)$-scaling of the wavelengths and the temperature. In the high-frequency limit, Wien's radiation law is still recovered, but not so the Rayleigh–Jeans law in the low-frequency regime, because $\rho(\nu \to 0) \sim 8\pi c^{-1}h^{-1}m\nu kT$, linear in frequency. Wien's displacement law is not valid either, as the peak of the distribution depends on the tachyon mass.

Acknowledgments

The author acknowledges the support of the Japan Society for the Promotion of Science, contract no P98239. Parts of this work were carried out at the Indian Institute of Astrophysics, Bangalore. I would like to thank Professor C V Vishveshwara for initiating my visit to the IIAP. An inspiring stay at the Institute of Mathematical Sciences, Madras, is likewise gratefully acknowledged.

Appendix A. Isometries in open RW cosmologies with negatively curved 3-space

The invariance group of the Poincaré half-space H^3 as defined in (2.2) is $SL(2,\mathbb{C})/\{\pm 1\}$, with the transitive group action [25, 36, 37]

$$(u,\xi) \to (|c\xi+d|^2+|c|^2u^2)^{-1}(u, (a\xi+b)\overline{(c\xi+d)}+a\bar{c}u^2), \tag{A.1}$$

for $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in SL(2,\mathbb{C})$. This action admits a continuous extension to the boundary at infinity of H^3; if we put $u=0$, then (A.1) reduces to the Möbius transformation $\xi \to (a\xi+b)(c\xi+d)^{-1}$ in the complex plane. We will make extensive use of the following subgroups of $SL(2,\mathbb{C})$ [38–40]:

$$\begin{aligned} R(\varphi) &= \begin{pmatrix} \mathrm{e}^{\mathrm{i}\varphi/2} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\varphi/2} \end{pmatrix}, \qquad & E(\vartheta) &= \begin{pmatrix} \cos(\vartheta/2) & -\sin(\vartheta/2) \\ \sin(\vartheta/2) & \cos(\vartheta/2) \end{pmatrix}, \\ T(\omega) &= \begin{pmatrix} 1 & \omega \\ 0 & 1 \end{pmatrix}, & H(\kappa) &= \begin{pmatrix} \kappa^{1/2} & 0 \\ 0 & \kappa^{-1/2} \end{pmatrix}, \end{aligned} \tag{A.2}$$

with complex ω and positive κ. $T(\omega)$ acts via (A.1) as translation group, $R(\varphi)$ as a group of rotations around the u-semiaxis and $H(\kappa)$ as a group of scale transformations,

$$\begin{aligned} &T(\omega)\colon (u,\xi) \to (u, \xi+\omega), \\ &R(\varphi)\colon (u,\xi) \to (u, \mathrm{e}^{\mathrm{i}\varphi}\xi), \\ &H(\kappa)\colon (u,\xi) \to \kappa(u,\xi). \end{aligned} \tag{A.3}$$

The action of $E(\vartheta)$ on H^3 is more complicated. It can be characterized as follows. (a) The point $(u=1, \xi=0)$ is a fixed point; (b) the half-plane $\xi_2=0$ is left invariant; (c) $E(\vartheta)$ maps the u-semiaxis into a Euclidean semicircle orthogonal to the complex plane with centre $(\xi_1=\cot\vartheta, \xi_2=0)$, and radius $|\sin\vartheta|^{-1}$, so that the two end points $u=0$ and ∞ of the u-semiaxis are mapped into $(-\tan(\vartheta/2), 0)$ and $(\cot(\vartheta/2), 0)$, respectively. If $\sin\vartheta=0$, then the u-semiaxis is mapped onto itself. (d) A tangent vector $|\boldsymbol{v}|(1,0,0)$ at the fixed point is mapped into $|\boldsymbol{v}|(\cos\vartheta, \sin\vartheta, 0)$.

Every element of $SL(2,\mathbb{C})$ admits a representation in terms of the subgroups (A.2) as

$$\begin{aligned} &\begin{pmatrix} a & b \\ c & d \end{pmatrix} = T(\omega)\,H(\kappa)\,R(\varphi)\,E(\vartheta)\,R(\psi), \\ &\omega\in\mathbb{C}, \qquad \kappa>0, \qquad 0\leqslant\varphi<2\pi, \qquad 0\leqslant\vartheta\leqslant\pi, \qquad 0\leqslant\psi<4\pi. \end{aligned} \tag{A.4}$$

Multiplying out the matrices in (A.4), we obtain

$$\begin{aligned} a &= \kappa^{1/2}\mathrm{e}^{\mathrm{i}(\varphi+\psi)/2}\cos(\vartheta/2) + \kappa^{-1/2}\omega\,\mathrm{e}^{-\mathrm{i}(\varphi-\psi)/2}\sin(\vartheta/2), \\ b &= -\kappa^{1/2}\mathrm{e}^{\mathrm{i}(\varphi-\psi)/2}\sin(\vartheta/2) + \kappa^{-1/2}\omega\,\mathrm{e}^{-\mathrm{i}(\varphi+\psi)/2}\cos(\vartheta/2), \\ c &= \kappa^{-1/2}\mathrm{e}^{-\mathrm{i}(\varphi-\psi)/2}\sin(\vartheta/2), \qquad d = \kappa^{-1/2}\mathrm{e}^{-\mathrm{i}(\varphi+\psi)/2}\cos(\vartheta/2). \end{aligned} \tag{A.5}$$

These equations can be inverted if $c\neq 0$, $d\neq 0$,

$$\begin{aligned} &\mathrm{e}^{\mathrm{i}\varphi/2} = \sqrt{|c||d|c^{-1}d^{-1}}, \qquad && \mathrm{e}^{\mathrm{i}\psi/2} = c|c|^{-1}\mathrm{e}^{\mathrm{i}\varphi/2}, \\ &\cos(\vartheta/2) = |d|\kappa^{1/2}, && \sin(\vartheta/2) = |c|\kappa^{1/2}, \\ &\kappa^{1/2} = (|c|^2+|d|^2)^{-1/2}, && \omega = c^{-1}(a-\bar{d}\kappa). \end{aligned} \tag{A.6}$$

The root defining $\mathrm{e}^{\mathrm{i}\varphi/2}$ must be chosen in such a way that $0\leqslant\varphi<2\pi$. It follows that the representation (A.4) is unique if $c\neq 0$, $d\neq 0$, i.e. there is only one choice of parameter values possible in the indicated ranges. Also note that one of the matrices $\pm\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ admits a representation (A.4) in which $0 \leqslant \psi < 2\pi$, as $R(\psi - 2\pi) = -R(\psi)$. If $c = 0$, we replace (A.4) by

$$\begin{pmatrix} a & b \\ 0 & a^{-1} \end{pmatrix} = T(\omega)\, H(\kappa)\, R(\psi), \tag{A.7}$$

with the parameter ranges as indicated in (A.4). We obtain, analogous to (A.6), $\mathrm{e}^{\mathrm{i}\psi/2} = a|a|^{-1}$, $\kappa^{1/2} = |a|$ and $\omega = ab$. Finally, if $d = 0$, we may use the parametrization

$$\begin{pmatrix} a & -c^{-1} \\ c & 0 \end{pmatrix} = T(\omega)\, H(\kappa)\, E(\pi)\, R(\psi), \tag{A.8}$$

with parameter ranges as in (A.4), and the inversion of (A.8) reads $\mathrm{e}^{\mathrm{i}\psi/2} = c|c|^{-1}$, $\kappa^{1/2} = |c|^{-1}$ and $\omega = ac^{-1}$. We may again restrict the ψ-range in (A.7) and (A.8) to $0 \leqslant \psi < 2\pi$ by replacing, if necessary, ψ by $\psi - 2\pi$. This affects only the overall sign of the matrix $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$, which does not enter in the action (A.1).

The isometrics (A.1) of H^3, complemented by $\tau \to \tau$, act via (2.4) and (2.3) on the forward lightcone as proper orthochronous Lorentz transformations. The application of (2.4) followed by (A.1) followed by (2.3) is just a realization of the isomorphism $SL(2,\mathbb{C})/\{\pm 1\} \sim SO^{+}(3,1)$; explicit formulae for the subgroups (A.2) are listed below.

Remark. The specification $a(\tau) = \tau$ in (2.1) is not essential in this appendix and in some other parts of this paper. For arbitrary scale factors, the line element on the forward lightcone is likewise obtained by applying (2.3) and (2.4) to (2.1); it is no longer Minkowskian, but still invariant under Lorentz transformations. For example, in the permeable spacetime discussed in [7] the Lorentz group stays the global symmetry group of the forward lightcone, both with respect to the metric and the permeability tensor.

Lorentz transformations are defined in this paper by $(t, x, y, z)^t \to L(t, x, y, z)^t$ (in contrast to $(t, x, y, z) \to (t, x, y, z)L$) with Lorentz matrices $L \in SO^{+}(3,1)$. We use in the following the shortcuts:

$$A_2(\alpha) := \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, \qquad A_3(\alpha) := \begin{pmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{pmatrix},$$
$$B_2(\beta) := \begin{pmatrix} \cosh\beta & \sinh\beta \\ \sinh\beta & \cosh\beta \end{pmatrix}. \tag{A.9}$$

The subgroups $R(\varphi)$ and $E(\vartheta)$ in (A.2) act in the forward lightcone as Lorentz transformations via the isomorphism mentioned after (A.8),

$$R(\varphi) \sim \begin{pmatrix} I_2 & 0_2 \\ 0_2 & A_2(\varphi) \end{pmatrix}, \qquad E(\vartheta) \sim \begin{pmatrix} 1 & \mathbf{0} & 0 \\ \mathbf{0} & A_2(\vartheta) & \mathbf{0} \\ 0 & \mathbf{0} & 1 \end{pmatrix}, \tag{A.10}$$

$$R(\pi/2)\, E(\vartheta)\, R(-\pi/2) = \begin{pmatrix} \cos(\vartheta/2) & -\mathrm{i}\sin(\vartheta/2) \\ -\mathrm{i}\sin(\vartheta/2) & \cos(\vartheta/2) \end{pmatrix} \sim \begin{pmatrix} 1 & \mathbf{0} \\ \mathbf{0} & A_3(\vartheta) \end{pmatrix}. \tag{A.11}$$

(By the way, $R(\varphi)\, E(\vartheta)\, R(\psi)$ parametrizes $SO(3)$, with the Eulerian angles in the range $0 \leqslant \varphi < 2\pi$, $0 \leqslant \psi < 2\pi$, $0 \leqslant \vartheta \leqslant \pi$.) The subgroup $H(\kappa)$ in (A.2) is realized in the lightcone as

$$H(\kappa) \sim \begin{pmatrix} B_2(\beta) & 0_2 \\ 0_2 & I_2 \end{pmatrix}, \qquad \cosh\beta := \tfrac{1}{2}(\kappa + \kappa^{-1}), \qquad \sinh\beta := \tfrac{1}{2}(\kappa - \kappa^{-1}). \tag{A.12}$$

The corresponding Lorentz transformations are just boosts along the x-axis, $t \to \hat{\gamma}(t - vx)$, $x \to \hat{\gamma}(x - vt)$, with $v := (1-\kappa^2)(1+\kappa^2)^{-1}$, $\hat{\gamma} := (1-v^2)^{-1/2}$, so that $\cosh\beta = \hat{\gamma}$, $\sinh\beta = -v\hat{\gamma}$ and $\kappa = (1-v)^{1/2}(1+v)^{-1/2}$.

Finally, we decompose the translations $T(\omega)$, $\omega = \omega_1 + \mathrm{i}\omega_2$, cf equation (A.2), into the commutative product $T(\omega_1)T(\mathrm{i}\omega_2)$. These subgroups admit the following representations in the lightcone:

$$T(\omega_1) \sim \begin{pmatrix} (2+\omega_1^2)/2 & -\omega_1^2/2 & \omega_1 & 0 \\ \omega_1^2/2 & (2-\omega_1^2)/2 & \omega_1 & 0 \\ \omega_1 & -\omega_1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{A.13}$$

which can in turn be split into the product

$$T(\omega_1) \sim \begin{pmatrix} 1 & \mathbf{0} & 0 \\ \mathbf{0} & A_2(\lambda) & \mathbf{0} \\ 0 & \mathbf{0} & 1 \end{pmatrix} \begin{pmatrix} B_2(\gamma) & 0_2 \\ 0_2 & I_2 \end{pmatrix} \begin{pmatrix} 1 & \mathbf{0} & 0 \\ \mathbf{0} & A_2(\mu) & \mathbf{0} \\ 0 & \mathbf{0} & 1 \end{pmatrix},$$
$$\cos\lambda = -\cos\mu = \omega_1(\omega_1^2+4)^{-1/2}, \qquad \sin\lambda = -\sin\mu = 2(\omega_1^2+4)^{-1/2},$$
$$\cosh\gamma = \tfrac{1}{2}(\omega_1^2+2), \qquad \sinh\gamma = \tfrac{1}{2}\omega_1(\omega_1^2+4)^{1/2}. \tag{A.14}$$

The same representation (A.13) also holds for $T(\mathrm{i}\omega_2)$, but with the y and z axes interchanged and ω_1 replaced by ω_2, so that

$$T(\mathrm{i}\omega_2) \sim \begin{pmatrix} 1 & \mathbf{0} \\ \mathbf{0} & A_3(\lambda) \end{pmatrix} \begin{pmatrix} B_2(\gamma) & 0_2 \\ 0_2 & I_2 \end{pmatrix} \begin{pmatrix} 1 & \mathbf{0} \\ \mathbf{0} & A_3(\mu) \end{pmatrix}, \tag{A.15}$$

with λ, μ and γ as in (A.14) ($\omega_1 \to \omega_2$). Neither $T(\omega_1)$ nor $T(\mathrm{i}\omega_2)$ are Lorentz boosts, as $A_{2,3}(\lambda)$ is not the inverse of $A_{2,3}(\mu)$.

Appendix B. Electromagnetic spectral waves in the forward lightcone

The spectral theory of the wave equation $A_{\mu;\alpha}{}^{;\alpha} = 0$, with gauge conditions $A_\mu{}^{;\mu} = 0$ and $A_0 = 0$, in a comoving RW frame with line element (2.1) was discussed in [41]. In the following we write $A_\mu = (A_0, A_u, A_{\xi_1}, A_{\xi_2}) = (0, \boldsymbol{A})$. Formulae (B.1)–(B.6) and their derivation can be found in [41] in a slightly different notation; we list them here to make this paper self-contained, and we specify the expansion factor as $a(\tau) = \tau$. The spectral elementary waves

$$\boldsymbol{A}_1^{\pm}(u,s) = \tau^{\mp\mathrm{i}s}(0,1,0)\,u^{\mathrm{i}s}, \qquad \boldsymbol{A}_2^{\pm}(u,s) = \tau^{\mp\mathrm{i}s}(0,0,-1)\,u^{\mathrm{i}s}, \tag{B.1}$$

constitute a complete orthogonal set of plane waves travelling along the u-semiaxis. s is a real spectral parameter. It is convenient to replace (B.1) by circularly polarized states,

$$\boldsymbol{A}^{R\pm}(u,s) := -2^{-1/2}(\boldsymbol{A}_1^{\pm} - \mathrm{i}\boldsymbol{A}_2^{\pm}), \qquad \boldsymbol{A}^{L\pm}(u,s) := -2^{-1/2}(\boldsymbol{A}_1^{\pm} + \mathrm{i}\boldsymbol{A}_2^{\pm}). \tag{B.2}$$

A complete set of elementary waves in H^3 is generated by applying the symmetry transformations

$$\alpha_\omega(u,\xi) := E(\pi)\,R(\pi)\,T(-\omega)(u,\xi) = (|\xi-\omega|^2 + t^2)^{-1}(u, \overline{\xi-\omega}) \tag{B.3}$$

to the plane waves (B.2), cf equation (A.1). We obtain in this way

$$\boldsymbol{A}^{R\pm}(u,\xi;\omega,s) = -2^{-1/2}\tau^{\mp\mathrm{i}s}(0,1,\mathrm{i})[\alpha'_\omega(u,\xi)]\,P^{\mathrm{i}s}(u,\xi;\omega)$$
$$= 2^{-1/2}\tau^{\mp\mathrm{i}s}\left(2u\overline{(\xi-\omega)}, \overline{(\xi-\omega)^2} - u^2, \mathrm{i}\,\overline{(\xi-\omega)^2} + \mathrm{i}u^2\right)\frac{P^{\mathrm{i}s}(u,\xi;\omega)}{(|\xi-\omega|^2+u^2)^2}, \tag{B.4}$$

$$\boldsymbol{A}^{L\pm}(u,\xi;\omega,s) = \overline{\boldsymbol{A}^{R\pm}(u,\xi;\omega,-s)}, \tag{B.5}$$

with the Poisson kernel

$$P(u,\xi;\omega) = \frac{u}{|\xi - w|^2 + u^2}. \tag{B.6}$$

Here $s \in \mathbb{R}$ and $\omega \in \mathbb{C}$ are spectral parameters, and $[\alpha'_\omega(u,\xi)]$ denotes the Jacobian of α_ω.

In the forward lightcone, a complete set of elementary waves $A_\mu^{R,L\pm}(t,\boldsymbol{x};\omega_1,\omega_2,s)$ is obtained by applying the coordinate transformation (2.4) onto the 4-vectors $(0,\boldsymbol{A}^{R,L\pm})$. The Jacobian of (2.4) reads

$$\frac{\partial(\tau,u,\xi_1,\xi_2)}{\partial(t,x_i)} = \begin{pmatrix} \frac{t}{(t^2-|\boldsymbol{x}|^2)^{1/2}} & \frac{-x^i}{(t^2-|\boldsymbol{x}|^2)^{1/2}} & & \\ \frac{y^2+z^2+x(x-t)}{(t-x)^2(t^2-|\boldsymbol{x}|^2)^{1/2}} & \frac{-(y^2+z^2+t(x-t))}{(t-x)^2(t^2-|\boldsymbol{x}|^2)^{1/2}} & \frac{-(y,z)}{(t-x)(t^2-|\boldsymbol{x}|^2)^{1/2}} & \\ \frac{-y}{(t-x)^2} & \frac{y}{(t-x)^2} & \frac{1}{t-x} & 0 \\ \frac{-z}{(t-x)^2} & \frac{z}{(t-x)^2} & 0 & \frac{1}{t-x} \end{pmatrix}, \tag{B.7}$$

and the spectral waves in the forward lightcone corresponding to (B.4) and (B.5) are calculated as

$$\begin{aligned} A_0^{R+} &= D^{-1}[2\omega_1 x + (1-\omega_1^2+\omega_2^2)y - 2\omega_1\omega_2 z + \mathrm{i}(-2\omega_2 x + 2\omega_1\omega_2 y - (1+\omega_1^2-\omega_2^2)z)], \\ A_1^{R+} &= D^{-1}[-2\omega_1 t + (1+\omega_1^2-\omega_2^2)y + 2\omega_1\omega_2 z + \mathrm{i}(2\omega_2 t - 2\omega_1\omega_2 y - (1-\omega_1^2+\omega_2^2)z)], \\ A_2^{R+} &= D^{-1}[-(1-\omega_1^2+\omega_2^2)t - (1+\omega_1^2-\omega_2^2)x + 2\omega_2 z \\ &\quad + \mathrm{i}(-2\omega_1\omega_2 t + 2\omega_1\omega_2 x + 2\omega_1 z)], \\ A_3^{R+} &= D^{-1}[2\omega_1\omega_2 t - 2\omega_1\omega_2 x - 2\omega_2 y + \mathrm{i}((1+\omega_1^2-\omega_2^2)t + (1-\omega_1^2+\omega_2^2)x - 2\omega_1 y)], \\ D &:= \sqrt{2}[(1+|\omega|^2)t + (1-|\omega|^2)x - 2\omega_1 y - 2\omega_2 z]^{2+\mathrm{i}s}, \qquad \omega =: \omega_1 + \mathrm{i}\omega_2, \end{aligned} \tag{B.8}$$

$$A_\mu^{R-} = (t^2 - |\boldsymbol{x}|^2)^{\mathrm{i}s} A_\mu^{R+}, \qquad A_\mu^{L\pm}(t,\boldsymbol{x};\omega_1,\omega_2,s) = \overline{A_\mu^{R\pm}(t,\boldsymbol{x};\omega_1,\omega_2,-s)}. \tag{B.9}$$

Each component of these 4-vectors satisfies the Euclidean Laplace equation, and the Lorentz condition is likewise met. Applying (2.4) onto (B.2), we obtain

$$\begin{aligned} A_0^{R+} &= 2^{-1/2}(y+\mathrm{i}z)(t-x)^{-2-\mathrm{i}s}, \qquad A_1^{R+} = -A_0^{R+}, \\ A_{2,3}^{R+} &= -2^{-1/2}(1,\mathrm{i})(t-x)^{-1-\mathrm{i}s}, \end{aligned} \tag{B.10}$$

$$A_\mu^{R-} = (t^2 - |\boldsymbol{x}|^2)\, A_\mu^{R+}, \qquad A_\mu^{L\pm}(t,\boldsymbol{x};s) = \overline{A_\mu^{R\pm}(t,\boldsymbol{x};-s)}. \tag{B.11}$$

These solutions complement (B.8) and (B.9) for $\omega = \infty$. Evidently, the waves (B.8) and (B.9) can be generated by applying to (B.10) and (B.11) the transformations $(t,\boldsymbol{x}) \rightarrow E(\pi)\,R(\pi)\,T(-\omega)\,(t,\boldsymbol{x})$, cf equations (B.3), (A.10), (A.14) and (A.15).

Although the metric on the forward lightcone is Minkowskian, the spectral elementary waves bear little resemblance to the exponentials encountered in a static Minkowski universe. In the individual, globally geodesic rest frames of galactic observers, the galactic background is radially receding, very much in contrast to a Minkowski universe. (The worldlines of galaxies emanate at $t = 0$ from the coordinate origin in these rest frames [1–3, 18].) Cosmic space is generated by this expanding galaxy grid, rather than by imaginary coordinate axes, and the spectral elementary waves are defined with respect to this grid, as is the energy of particles. The galactic recession generates a time asymmetry, so that the wavefronts of positive and negative frequency solutions (which here means positive/negative s) are very differently structured.

We discuss at first the geometry and the evolution of the wavefronts in comoving coordinates. The phase of the plane waves (B.4) and (B.5) is of course the same for left and right polarization,

$$\psi(\tau, u, \xi; s, \omega) = s \log(\tau^{\mp 1} P(u, \xi; \omega)), \tag{B.12}$$

with the upper (lower) sign valid for positive (negative) s. The phase coincides with the general solution of the eikonal equation, $g^{\mu\nu}\psi_{,\mu}\psi_{,\nu} = 0$, which means that the eikonal approximation is exact [17]. The elementary waves propagate with frequency/energy $\omega = \lambda\!\!\!^{-}{}^{-1} = -\psi_{,\tau} = |s|\tau^{-1}$. ($\omega$ should not be confused with the complex spectral variable in the Poisson kernel; in the following we will denote frequencies by $\omega^{\pm}$.) The sign of the phase in (B.12) is chosen in a way such that the photon energy is positive. The surfaces of constant phase, $\tau^{\mp 1} P(u, \xi; \omega) = (2\beta)^{-1}$, $\beta > 0$, are horospheres [25, 36],

$$(u - \beta\tau^{\mp 1})^2 + |\xi - \omega|^2 = \beta^2\tau^{\mp 2}, \tag{B.13}$$

i.e. Euclidean spheres of radius $\beta\tau^{\mp 1}$ emanating from $\xi = \omega$ on the boundary at infinity of H^3. (The upper sign in the exponents always refers to positive s.) The waves (B.1) and (B.2) travel along the u-semiaxis; their wavefronts are obtained by putting $P = u$ in (B.12), so that the spheres (B.13) are replaced by horoplanes

$$u = (2\beta)^{-1}\tau^{\pm 1}, \qquad \beta > 0. \tag{B.14}$$

Next we calculate the rays orthogonal to the wavefronts (B.13). Rays are defined by equating the line element (2.1) (with $a(\tau) = \tau$) to zero. Along the u-semiaxis, a ray is given either by $u = \alpha\tau$ or $u = (\alpha\tau)^{-1}$, with an integration constant $\alpha > 0$. The ray bundle emanating from $\xi = \omega$ on the boundary of H^3 is constructed by applying to the rays ($u = \alpha\tau, \xi = 0$) the symmetry transformation (A.1), with

$$\begin{aligned} \begin{pmatrix} a & b \\ c & d \end{pmatrix} &= T(\omega)\, R(\varphi)\, H(\kappa)\, T(1)\, E(\pi/2), \\ a &= 2^{1/2}\kappa^{1/2} \exp(\mathrm{i}\varphi/2) + 2^{-1/2}\omega\kappa^{-1/2} \exp(-\mathrm{i}\varphi/2), \\ c = d &= 2^{-1/2}\kappa^{-1/2} \exp(-\mathrm{i}\varphi/2), \qquad b = c\omega. \end{aligned} \tag{B.15}$$

$E(\pi/2)$ maps the u-semiaxis into a semicircle of Euclidean radius one, orthogonal to the complex plane, so that ($u = 0, \xi = 0$) is mapped into ($u = 0, \xi = -1$) and ($u = \infty, \xi = 0$) into ($u = 0, \xi = 1$), see after (A.3). This semicircle is shifted along the ξ_1-axis by $T(1)$, so that ($u = 0, \xi = -1$) is mapped into the origin. $H(\kappa)$ is a scale transformation, $R(\varphi)$ rotates the semicircle around the u-semiaxis, and $T(\omega)$ finally shifts its initial point into ω.

Applying (A.1) and (B.15) onto ($u = \alpha\tau, \xi = 0$), we obtain all rays emanating from $\xi = \omega$,

$$u(\tau) = \frac{2\kappa\alpha\tau}{1 + \alpha^2\tau^2}, \tag{B.16}$$

$$\xi^-(\tau) = \omega + \frac{2\kappa\alpha^2\tau^2 \exp(\mathrm{i}\varphi)}{1 + \alpha^2\tau^2}. \tag{B.17}$$

The rays of this bundle are labelled by parameters $\kappa, \alpha > 0$ and $0 \leqslant \varphi < 2\pi$. Rays with ω as the terminal point (for $\tau \to \infty$) are obtained by applying (B.15) onto ($u = (\alpha\tau)^{-1}, \xi = 0$). This amounts to replacing $\alpha\tau$ by $(\alpha\tau)^{-1}$ in (B.16) and (B.17), so that $u(\tau)$ remains unchanged, and the complex coordinate of these rays reads

$$\xi^+(\tau) = \omega + \frac{2\kappa \exp(\mathrm{i}\varphi)}{1 + \alpha^2\tau^2}. \tag{B.18}$$

By construction, the rays (B.16)–(B.18) identically satisfy (B.13), i.e.

$$(u(\tau) - \beta_\pm \tau^{\mp 1})^2 + |\xi^\pm(\tau) - \omega|^2 = \beta_\pm^2 \tau^{\mp 2}, \qquad \beta_\pm := \kappa \alpha^{\mp 1}, \tag{B.19}$$

and along these rays we have the wavevector

$$\begin{aligned} \boldsymbol{k}_i(\tau) &= \partial\psi/\partial(u,\xi) = s\kappa^{-1}[\sinh(\log(\alpha\tau)^{\mp 1}), -\exp(\mathrm{i}\varphi)], \\ \boldsymbol{k}^i(\tau) &= \tau^{-1}|s|\boldsymbol{v}_\pm^i, \qquad \boldsymbol{v}_\pm^j = (\mathrm{d}u/\mathrm{d}\tau, \mathrm{d}\xi^\pm/\mathrm{d}\tau), \end{aligned} \tag{B.20}$$

attached. ($\partial\psi/\partial(u,\xi)$ is evaluated along the trajectories $(u(\tau), \xi^\pm(\tau))$, with ξ^+ for positive s.) The photon momentum reads of course $\boldsymbol{p}^i = \hbar\boldsymbol{k}^i$, In addition to (B.16)–(B.18), there are the solutions $(u = (\alpha\tau)^{\mp 1}, \xi = \omega)$ of $\mathrm{d}s^2 = 0$, cf equation (2.1). These trajectories evidently solve (B.19) with $\beta_\pm = \alpha^{\mp 1}/2$.

Inserting (2.4) into (B.12), we obtain the eikonal in the forward lightcone,

$$\psi^\pm = \pm|s|\log[(t^2 - |\boldsymbol{x}|^2)^{\mp 1/2} P(t, \boldsymbol{x}; \omega_1, \omega_2)], \tag{B.21}$$

$$P(t, \boldsymbol{x}; \omega_1, \omega_2) := \frac{\sqrt{t^2 - |\boldsymbol{x}|^2}}{(1 + |\omega|^2)(t + \boldsymbol{n}\boldsymbol{x})}, \qquad \boldsymbol{n} := \frac{1}{1 + |\omega|^2}(1 - |\omega|^2, -2\omega_1, -2\omega_2), \tag{B.22}$$

with $\omega = \omega_1 + \mathrm{i}\omega_2$. The upper (lower) sign again refers to positive (negative) s. This is the phase of the spectral waves (B.8) and (B.9). If we put $P = u$ in (B.12), we recover via (2.4) the phase of the waves (B.10) and (B.11),

$$\psi^\pm = \pm|s|\log\frac{(t^2 - |\boldsymbol{x}|^2)^{(1\mp 1)/2}}{t - x}. \tag{B.23}$$

The wavefronts $\tau P = (2\beta)^{-1}$, cf equation (B.13), read in the forward lightcone

$$|\boldsymbol{x} - \boldsymbol{m}|^2 = (t - |\boldsymbol{m}|)^2, \qquad \boldsymbol{m} := -\boldsymbol{n}(4\beta)^{-1}(1 + |\omega|^2), \tag{B.24}$$

with $\boldsymbol{n}$ as in (B.22). They are the surfaces of constant eikonal ψ^-, cf equation (B.21), and belong to the waves (B.9). Only if $t > |\boldsymbol{m}|$, do these spheres lie in the lightcone (and touch its boundary, $t = |\boldsymbol{x}|$, tangentially). The wavefronts $\tau^{-1}P = (2\beta)^{-1}$ (corresponding to positive spectral variable s and constant eikonal ψ^+) are planes in the lightcone,

$$\boldsymbol{n}\boldsymbol{x} = 2\beta(1 + |\omega|^2)^{-1} - t, \tag{B.25}$$

and belong to the spectral waves (B.8).

If $P = u$, we have for negative s the wavefronts $\tau u = (2\beta)^{-1}$ in comoving coordinates, cf equation (B.14), which read in the lightcone as

$$(x - (4\beta)^{-1})^2 + y^2 + z^2 = (t - (4\beta)^{-1})^2. \tag{B.26}$$

They are the surfaces of constant phase ψ^-, cf equation (B.23), of the modes (B.11). Analogous to (B.24), these spheres lie in the lightcone only if $t > (4\beta)^{-1}$. Finally, the wavefronts $\tau^{-1}u = (2\beta)^{-1}$ corresponding to constant ψ^+ in (B.23), are planes in the lightcone,

$$x = t - 2\beta, \tag{B.27}$$

and belong to the modes (B.10).

In the following we partition the light rays into bundles, label them by means of wavefronts, and define the energy of photons on the individual rays. We determine at first the rays orthogonal to the wavefronts (B.24)–(B.27). The rays $(u(\tau), \xi^-(\tau))$, cf (B.16) and (B.17), are mapped by (2.4) into the lightcone; we obtain

$$x = \frac{|\tilde{\omega}|^2 - 1}{|\tilde{\omega}|^2 + 1} t + a, \qquad y + \mathrm{i}z = \frac{2\tilde{\omega}}{|\tilde{\omega}|^2 + 1} t - a\tilde{\omega} - \frac{1}{\alpha}\exp(\mathrm{i}\varphi),$$
$$\tilde{\omega} := \omega + 2\kappa\exp(\mathrm{i}\varphi), \qquad |\tilde{\omega}|^2 = |\omega|^2 + 4\kappa(\kappa + \mathrm{Re}(\omega \mathrm{e}^{-\mathrm{i}\varphi})), \tag{B.28}$$
$$a := \frac{2}{(|\tilde{\omega}|^2 + 1)\alpha}(\kappa - \mathrm{Re}(\tilde{\omega}\,\mathrm{e}^{-\mathrm{i}\varphi})).$$

These rays identically satisfy (B.24) if we put $\beta = \kappa\alpha$, as well as

$$(1 + |\omega|^2)(t + \boldsymbol{n}\boldsymbol{x}) = \frac{2\kappa}{(1 + |\tilde{\omega}|^2)\alpha}(4\kappa\alpha t - 1 - |\omega|^2). \tag{B.29}$$

They are the orthogonal rays to the spherical wavefronts (B.24), originating at

$$\begin{pmatrix} t \\ x \end{pmatrix} = \frac{1}{4\alpha\kappa}(|\omega|^2 \pm 1), \qquad y + \mathrm{i}z = \frac{\omega}{2\alpha\kappa}. \tag{B.30}$$

The frequencies of the waves (B.9) read, cf equation (B.21),

$$\omega^-(t, \boldsymbol{x}) = -\psi^-_{,t} = \frac{|s|(t^2 + |\boldsymbol{x}|^2 + 2t\boldsymbol{n}\boldsymbol{x})}{(t^2 - |\boldsymbol{x}|^2)(t + \boldsymbol{n}\boldsymbol{x})} = \frac{|s|(4\beta t - 1 - |\omega|^2)}{(1 + |\omega|^2)(t + \boldsymbol{n}\boldsymbol{x})}. \tag{B.31}$$

(The last equality holds on the wavefronts (B.24).) Along the rays (B.28), this gives

$$\omega^- = |s|\alpha(2\kappa)^{-1}(1 + |\tilde{\omega}|^2), \tag{B.32}$$

where we made use of (B.29). The frequency ω^- is time independent and determined by the spectral variables s and ω in the phase ψ^-, cf equation (B.21), as well as by the integration parameters α, κ and φ, which label the rays of the bundle (B.28). Photons on individual rays are defined by the propagating wavefronts (B.24) and the polarization of the corresponding spectral mode (B.9), and they carry the energy ω^-.

The trajectories $(u(\tau), \xi^+(\tau))$, cf equation (B.16) and (B.18), read in the lightcone as

$$x = \frac{|\omega|^2 - 1}{|\omega|^2 + 1} t + b, \qquad y + \mathrm{i}z = \frac{2\omega}{|\omega|^2 + 1} t - b\omega + \frac{1}{\alpha}\exp(\mathrm{i}\varphi),$$
$$b := \frac{2}{(|\omega|^2 + 1)\alpha}(\kappa + \mathrm{Re}(\omega\,\mathrm{e}^{-\mathrm{i}\varphi})), \qquad t > \frac{|\omega|^2 + 1}{2}\left(\frac{1}{2\alpha\kappa} + b\right). \tag{B.33}$$

This bundle comprises the orthogonal rays of the modes (B.8), as they identically satisfy (B.25) (with $\beta = \kappa\alpha^{-1}$). The photon frequencies along these rays are readily calculated from (B.21) and (B.25),

$$\omega^+(t, \boldsymbol{x}) = -\psi^+_{,t} = |s|(t + \boldsymbol{n}\boldsymbol{x})^{-1}, \qquad \omega^+ = |s|\alpha(2\kappa)^{-1}(1 + |\omega|^2). \tag{B.34}$$

The rays $(u = (\alpha\tau)^{-1}, \xi = \omega)$, read in the lightcone as

$$x = \frac{|\omega|^2 - 1}{|\omega|^2 + 1} t + \frac{1}{\alpha(|\omega|^2 + 1)}, \qquad y + \mathrm{i}z = \frac{\omega}{|\omega|^2 + 1}\left(2t - \frac{1}{\alpha}\right). \tag{B.35}$$

They emerge from $(t = x = (2\alpha)^{-1}, y + \mathrm{i}z = 0)$, the centre of the spherical wavefronts (B.26), if we identify $\beta = \alpha/2$, and they identically solve (B.26). We obtain from (B.23)

$$\omega^-(t, \boldsymbol{x}) = \frac{|s|(t^2 + |\boldsymbol{x}|^2 - 2tx)}{(t^2 - |\boldsymbol{x}|^2)(t - x)}, \qquad \omega^- = |s|\alpha(1 + |\omega^2|), \tag{B.36}$$

for the frequencies of the elementary waves (B.11) and for the photon energy on the rays of the orthogonal bundle (B.35).

Finally, the rays ($u = \alpha\tau, \xi = \omega$) are mapped by (2.4) onto

$$x = t - \alpha^{-1}, \qquad y + \mathrm{i}z = \alpha^{-1}\omega, \qquad t > (2\alpha)^{-1}(|\omega|^2 + 1), \tag{B.37}$$

and they solve (B.27) if we put there $\beta = (2\alpha)^{-1}$. The frequencies of the elementary waves (B.10) and the photon energy along their orthogonal trajectories (B.37) read, analogously to (B.36),

$$\omega^+(t, x) = |s|(t - x)^{-1}, \qquad \omega^+ = |s|\alpha. \tag{B.38}$$

The ray bundles (B.28), (B.33), (B.35) and (B.37) comprise all possible photon worldlines, and equations (B.31), (B.34), (B.36) and (B.38) define the respective photon energy in the linearly expanding galaxy grid.

References

[1] Milne E A 1932 *Nature* **130** 9
[2] Milne E A 1935 *Relativity, Gravitation and World-Structure* (Oxford: Clarendon)
[3] Milne E A 1948 *Kinematic Relativity* (Oxford: Clarendon)
[4] Robertson H P 1938 *Z. Astrophys.* **15** 69
[5] Robertson H P and Noonan T W 1968 *Relativity and Cosmology* (Philadelphia, PA: Saunders)
[6] Tomaschitz R 1998 *Int. J. Mod. Phys.* D **7** 279
[7] Tomaschitz R 1998 *Int. J. Theor. Phys.* **37** 1121
[8] Tomaschitz R 1998 *Astrophys. Space Sci.* **259** 255
[9] Sandage A 1961 *Astrophys. J.* **133** 355
[10] Dyson F J 1979 *Rev. Mod. Phys.* **51** 447
[11] Sandage A 1988 *Ann. Rev. Astron. Astrophys.* **26** 561
[12] Feinberg G 1967 *Phys. Rev.* **159** 1089
[13] Tanaka S 1961 *Prog. Theor. Phys.* **4** 171
[14] Terletsky Ya P 1961 *Sov. Phys.–Dokl.* **5** 782
[15] Sommerfeld A 1904 *Proc. Konink. Akad. Wet. (Amsterdam) Section of Sciences* **7** 346
[16] Anderson R, Vetharaniam I and Stedman G E 1998 *Phys. Rep.* **295** 93
[17] Hurt N E 1983 *Geometric Quantization in Action* (Dordrecht: Reidel)
[18] Infeld L and Schild A 1945 *Phys. Rev.* **68** 250
[19] Fulling S A, Parker L and Hu B L 1974 *Phys. Rev.* D **10** 3905
[20] Davies P C W and Fulling S A 1977 *Proc. R. Soc.* A **354** 59
[21] Tanaka T and Sasaki M 1997 *Phys. Rev.* D **55** 6061
[22] Gibbons G W 1979 Quantum field theory in curved spacetime *General Relativity: an Einstein Centenary Survey* ed S W Hawking and W Israel (Cambridge: Cambridge University Press)
[23] Birrell N D and Davies P C W 1982 *Quantum Fields in Curved Space* (Cambridge: Cambridge University Press)
[24] Ahlfors L V 1981 *Möbius Transformations in Several Dimensions* (*University of Minnesota Lecture Notes*)
[25] Beardon A F 1983 *The Geometry of Discrete Groups* (New York: Springer)
[26] Tomaschitz R 1996 *Chaos Solitons Fractals* **7** 753
[27] Wentzel G 1949 *Quantum Theory of Fields* (New York: Interscience)
[28] Goldhaber A S and Nieto M M 1971 *Rev. Mod. Phys.* **43** 277
[29] Dirac P A M 1973 Fundamental constants and their development in time *The Physicist's Conception of Nature* ed J Mehra (Dordrecht: Reidel)
[30] Dyson F J 1972 The fundamental constants and their time variation *Aspects of Quantum Theory* ed A Salam and E P Wigner (Cambridge: Cambridge University Press)
[31] Tolman R C 1934 *Relativity, Thermodynamics and Cosmology* (Oxford: Clarendon)
[32] Tomaschitz R 1993 *J. Math. Phys.* **34** 1022
[33] Mashhoon B 1973 *Phys. Rev.* D **8** 4297
[34] Tomaschitz R, in preparation
[35] Tomaschitz R 1999 *Int. J. Mod. Phys* A at press
[36] Ratcliffe J G 1994 *Foundations of Hyperbolic Manifolds* (New York: Springer)

[37] Matsuzaki K and Taniguchi M 1998 *Hyperbolic Manifolds and Kleinian Groups* (Oxford: Clarendon)
[38] Ford L 1951 *Automorphic Functions* (Bronx: Chelsea)
[39] Naimark M A 1964 *Linear Representations of the Lorentz Group* (Oxford: Pergamon)
[40] Lehner J 1964 *Discontinuous Groups and Automorphic Functions* (Providence, RI: American Mathematical Society)
[41] Tomaschitz R 1993 *J. Math. Phys.* **34** 3133